State Change in Coincident Indicators

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Abstract

This paper statistically investigates whether validity of a two-state Markov switching model depends on sample periods. The main findings are as follows. First, we find that it depends on sample periods whether the one-state or the two-state model is valid. In the recent sample period after 1991, five out of the ten indicators are amenable to the two-state model. Second, the two-state model is compatible with the indicators related to production and final users' purchasing decision in the recent sample period. Third, the one-state model is better suited for the intermediate transactions and the shipments of goods with durability such as investment goods and consumer durables.

Key words: Likelihood ratio test, Markov switching, Coincident indicators

JEL classification: C12, C24, E32

1 Introduction

A large part of empirical research in macroeconomics is devoted to finding statistical models that give a good description of time series data. Various models are introduced and tested (see Canova, 2007; Hamilton, 1994, for example). Among them, the Markov Switching model, introduced by Hamilton (1989), and its variants are most frequently used in economics and finance. These models attempt to explain dynamics in time series by state changes governed by an unobservable Markov chain. However, it is still arguable whether these models are appropriate to explain the economic time series.

Hamilton (1996) proposes Lagrange-multiplier specification tests for a variety of forms of autocorrelation, generalized ARCH effects, Markovian dynamics, and omitted explanatory variables for both the mean and the variance. These tests are implemented in Engel and Hamilton (1990). But, we cannot use them to test the null hypothesis of a one-state model, such as autoregressive models, against the alternative hypothesis of a multi-state model, such as the two-state Markov switching model.

As Hansen (1992) pointed out, a difficulty arises from the fact that some parameters are unidentified under the null hypothesis. This is the problem of unidentified nuisance parameters, which was analyzed by Davies (1977, 1987). When the nuisance parameters exist under the null hypothesis, the standard asymptotic theory cannot be applied and the distribution under the null hypothesis is unknown. Further, the information matrix is singular because the underlying states are not observable and the likelihood function has multiple local optima and some of the elements of the score vector (the first-order derivatives) are identically zeros under the null hypothesis.

Hansen (1996) extended the empirical process theory to a wide class of estimation problems and test statistics. Hansen (1992) used it to derive a bound for the asymptotic distribution of the standardized likelihood ratio statistic for Markov switching models, and found that a one-state model is not rejected against the two-state Markov switching model. Garcia (1998) examined the asymptotic distribution of the likelihood-ratio test statistic, assuming the score is not identically zero under the null hypothesis, and also found no evidence for the two-state Markov model. But, the power of such a test is unknown.

Carrasco, Hu, and Ploberger (2014) extended the information matrix test of White (1982) to allowing the parameters to follow flexible weakly dependent processes. Their test cannot reject the null of a one-state constant mean against the alternative of the two states for Hamilton (1989)'s data. However, they found that the constant-mean hypothesis is rejected with the extended data up to the fourth quarter (Q4) of 2010 against the alternative of the switching in both mean and variance.

Finally, Cho and White (2007) proposed a *quasi*-likelihood ratio (QLR) test and derived its asymptotic null distribution. Qu and Zhuo (2021) used a higher-order approximation to refine the asymptotic null distribution of the QLR statistic. They found strong evidence for the Markov switching specification when they applied their methods to US quarterly GDP growth-rate data from the first quarter (Q1) of 1960 to the fourth quarter (Q4) of 2006 or 2014.

This paper investigates whether validity of a two-state Markov switching model depends on sample periods. We apply the standardized likelihood-ratio (LR) test proposed by Hansen (1992) to the coincident indicators of Japan at monthly frequency. In the literature, the computational burden deters researchers from using the standardized LR statistic. In addition, the coincident indicators are complied to understand the business cycle nature of the economy, but, to the best of my knowledge, they are rarely used in the literature. We attempt to fill these gaps as much as possible.

The main findings are as follows. First, we find that it depends on sample periods whether the one-state or the two-state model is valid. In the recent sample period after 1991, five out of the ten indicators are amenable to the two-state model. Second, the two-state model is compatible with the indicators related to production and final users' purchasing decision in the recent sample period. Third, the one-state model is better suited for the intermediate transactions and the shipments of goods with durability such as investment goods and consumer durables.

The rest of the paper is organized as follows. In section 2, we briefly discuss the statistical model and the testing hypothesis to be studied, reviewing the likelihood ratio test statistic proposed by Hansen (1992). Section 3 presents empirical results based on the time series data, the coincident indicators of Japan. The final section is allocated to discussion.

2 Markov Switching Model and Test Statistic

2.1 Markov Switching Model

In some experiments with our monthly data, we found that generalized models do not give numerically normal convergence. Thus, we focus on a simple Markov-switching model, which is used in Cecchetti, Lam, and Mark (1990) and studied by Hansen (1992) and Qu and Zhuo (2021). Suppose we have two different states of economy. Let y_t an economic time-series variable at time t $(t=1, 2, \cdots, n)$. Then, the simple Markov switching model that we use is given as follows:

$$y_t=\mu+\mu_d S_t+\sum_{j=1}^{J}\phi_j y_{t-j}+\varepsilon_t, \tag{1}$$

$$\varepsilon_t \sim N(0, \sigma^2), \tag{2}$$

where the constant parameters of μ, μ_d, ϕ_j and σ are all unknown, and S_t denotes the economic state at time t, taking either 1 or 0. The transition between the states is assumed to be governed by a first-order Markov process that is independent of ε_t. Then, the transition probabilities are:

$$P[S_t=1|S_{t-1}=1]=p, \tag{3}$$

$$P[S_t=0|S_{t-1}=0]=q. \tag{4}$$

When μ_d is zero, this model reduces to a one-state model. Therefore, it is interesting to conduct the following hypothesis testing to see if the two-state model is appropriate.

$$H_0 : \mu_d = 0, \quad H_1 : \mu_d \neq 0, \tag{5}$$

where H_0 denotes the null hypothesis and H_1 the alternative one. One might use the conventional t-statistic to test H_0, but it does not have the standard null distribution. Neither do other conventional statistics, such as the likelihood ratio or the chi-square statistics. This is because p and q are unidentified under the null hypothesis. That is, we cannot find unique estimates for these parameters to maximize the likelihood function. Further, the scores with respect to μ_d, p and q are identically zeros under the null hypothesis. Then, the standard distributional theory is inapplicable.

To circumvent these problems, Hansen (1992) proposed a standardized likelihood ratio test statistic and resorted to Monte Carlo simulations to compute its p-values. Setting $J=4$ in eq. (1), Hansen (1992) found that the Markov switching specification is ‘unlikely’ for the Hamilton (1989)’s US real GNP data from 1954Q2 to 1984Q4. To the contrary, Qu and Zhuo (2021) gave strong evidence favoring the Markov switching specification with $J=1$ and US real GDP data during the periods of 1960Q1 to 2006Q4 and 1960Q1 to 2014Q4. We set $J=1$ in the later analysis.

2.2 The Standardized Likelihood Ratio Test Statistic

We provide a schematic explanation for the standardized likelihood ratio test statistic proposed by Hansen (1992). Let $l_t(\cdot)$ a log-transformed probability density of the model consisting of eq.(1) to eq.(4). Then, the log-likelihood function can be written in the form:

$$\mathcal{L}_n(\alpha, \theta) = \sum_{t=1}^{n} l_t(\alpha, \theta), \tag{6}$$

where n is the sample size, $\alpha=(\mu_d, p, q)$ and $\theta=(\mu, \phi_1, \sigma)$. Note that θ is identified under H_0 in (5). Suppose that, in a large sample, the pseudo-true value of θ for fixed values of α can be written as:

$$\theta(\alpha)=\underset{\theta}{\operatorname{argmax}}\lim_{n\to\infty}\frac{1}{n}E\mathcal{L}_n(\alpha, \theta). \tag{7}$$

Then, the concentrated log-likelihood function is given by

$$\mathcal{L}_n^C(\alpha)=\mathcal{L}_n(\alpha, \theta(\alpha)). \tag{8}$$

To test H_0 in (5), consider the following likelihood ratio function:

$$LR_n(\alpha_A)=\mathcal{L}_n^C(\alpha_A)-\mathcal{L}_n^C(\alpha_N), \tag{9}$$

where α_N is α under H_0 and α_A is α under H_1. Hansen (1992) worked on the test statistic given by the supremum of eq.(9):

$$LR_n=\sup_{\alpha_A}LR_n(\alpha_A). \tag{10}$$

To study a bound of the asymptotic distribution of this statistic, consider the following decomposition for any α:

$$LR_n(\alpha)=E[LR_n(\alpha)]+Q_n(\alpha), \tag{11}$$

where $E[LR_n(\alpha)]$ is the mean and $Q_n(\alpha)$ is the deviation from the mean. Note:

$$Q_n(\alpha)=\sum_{t=1}^{n}q_t(\alpha), \tag{12}$$

where

$$q_t(\alpha)=l_t(\alpha, \theta(\alpha))-l_t(\alpha_N, \theta(\alpha_N)) \\ -E[l_t(\alpha, \theta(\alpha))-l_t(\alpha_N, \theta(\alpha_N))]. \tag{13}$$

Under an empirical process central limit theorem, the $Q_n(\alpha)$ weakly converges to a mean zero Gaussian process, $Q(\alpha)$:

$$\frac{1}{\sqrt{n}} Q_n(\alpha) \Longrightarrow Q(\alpha), \tag{14}$$

as $n \to \infty$. For different values of α, the covariance function of $Q(\alpha)$ is given by

$$K(\alpha_1, \alpha_2) = \sum_{k=-\infty}^{\infty} E[q_t(\alpha_1) q_{t+k}(\alpha_2)], \tag{15}$$

and the associated variance function is

$$V(\alpha) = K(\alpha, \alpha). \tag{16}$$

Although we do not know the value of the mean function in eq.(11), we know from eq.(9) that, under the H_0, it takes values less than and equal to zero:

$$E[LR_n(\alpha)] \leq 0. \tag{17}$$

This implies

$$\frac{1}{\sqrt{n}} LR_n(\alpha) \leq \frac{1}{\sqrt{n}} Q_n(\alpha). \tag{18}$$

As assumed before, the right-hand side weakly converges to $Q(\alpha)$. Therefore, taking sup of the both sides in eq.(18), we can obtain a bound for the asymptotic distribution of the statistic in eq.(10). That is,

$$LR_n \leq \sup_{\alpha_A} \frac{1}{\sqrt{n}} Q_n(\alpha_A) \Longrightarrow \sup_{\alpha_A} Q(\alpha_A). \tag{19}$$

Now, let $\hat{\theta}_n(\alpha)$ the maximum-likelihood (ML) estimator of $\theta(\alpha)$ in eq.(7):

$$\hat{\theta}_n(\alpha) = \max_{\theta} \mathcal{L}_n(\alpha, \theta), \tag{20}$$

for fixed values of α. It is assumed that $\hat{\theta}_n(\alpha)$ is consistent for $\theta(\alpha)$ at the rate $\sqrt{n}$, uniformly in α. Further, we assume that we have a stochastic order relation for the following Euclidean metric with $\hat{\theta}(\alpha)$, the ML estimates of θ for fixed α:

$$\sup_{\alpha} \| \mathcal{L}_n^C(\alpha,\ \theta(\alpha)) - \mathcal{L}_n^C(\alpha,\ \hat{\theta}(\alpha)) \| = O_p(1). \tag{21}$$

Let $\hat{q}_t(\alpha)$ the estimator of $q_t(\alpha)$ in eq.(13) associated with $\hat{\theta}(\alpha)$, and it is used to compute $\hat{Q}_n(\alpha)$ via eq.(12). Then, eq.(21) implies

$$\sup_{\alpha} \| Q_n(\alpha) - \hat{Q}_n(\alpha) \| = O_p(1). \tag{22}$$

With these consistent estimators, the sample analogue of eq. (15) can be written as

$$\hat{K}_n(\alpha_1,\ \alpha_2) = \sum_{t=1}^{n} \hat{q}_t(\alpha_1)\hat{q}_t(\alpha_2) + \sum_{k=1}^{M} w_{kM} \sum_{t=k+1}^{n} (\hat{q}_{t-k}(\alpha_1)\hat{q}_t(\alpha_2) + \hat{q}_t(\alpha_1)\hat{q}_{t-k}(\alpha_2)), \tag{23}$$

where w_{kM} is the Bartlett kernel with a bandwidth of M:

$$w_{kM} = 1 - \frac{k}{M+1}. \tag{24}$$

Newey and West (1987) provides the property of this estimator in detail. When we set $\alpha_1 = \alpha_2$ in eq.(23) and divide its both sides by n to obtain the estimator of variance: $\hat{V}_n(\alpha)/n$.

For the sample analogue of eq.(18) with the estimator $\hat{\theta}(\alpha)$, we have,

$$\frac{1}{\sqrt{n}} \widehat{LR_n}(\alpha) \leq \frac{1}{\sqrt{n}} \hat{Q}_n(\alpha). \tag{25}$$

The right-hand side weakly converges to $Q(\alpha)$ under the conditions of eq.(14) and eq.(22).

Hansen (1992) argued that the bound has excessively strong tendency not to reject the null hypothesis: it is too conservative in practice. To reduce the over-conservative tendency, it is proposed to standardize the statistic so that the variance is same for all values of α. The standardized version is obtained by dividing both sides by $\sqrt{\hat{V}_n(\alpha)/n}$:

$$\frac{\widehat{LR_n}(\alpha)}{\sqrt{\hat{V}_n(\alpha)}} \leq \frac{\hat{Q}_n(\alpha)}{\sqrt{\hat{V}_n(\alpha)}}. \tag{26}$$

Hansen (1992) assumes:

$$\frac{\hat{Q}_n(\alpha)}{\sqrt{\hat{V}_n(\alpha)}} \Longrightarrow \frac{Q(\alpha)}{\sqrt{V(\alpha)}}. \tag{27}$$

Define the standardized likelihood ratio function as

$$\widehat{LR_n^*}(\alpha) = \frac{\widehat{LR_n}(\alpha)}{\sqrt{\hat{V}_n(\alpha)}}. \tag{28}$$

Then, the standardized likelihood ratio statistic is

$$\widehat{LR_n^*} = \sup_{\alpha} \widehat{LR_n^*}(\alpha). \tag{29}$$

Similarly, we define

$$\hat{Q}_n^*(\alpha) = \frac{\hat{Q}_n(\alpha)}{\sqrt{\hat{V}_n(\alpha)}}, \tag{30}$$

and

$$Q^*(\alpha) = \frac{Q(\alpha)}{\sqrt{V(\alpha)}}. \tag{31}$$

Together with eq.(26) and eq.(27), we have

$$\widehat{LR_n^*} \leq \sup_{\alpha_A} \hat{Q}_n^*(\alpha_A) \Longrightarrow \sup_{\alpha_A} Q^*(\alpha_A). \tag{32}$$

Finally, we have the following bound for the asymptotic distribution of the statistic in eq.(29):

$$\begin{aligned} P\left\{\widehat{LR_n^*} \geq x\right\} &\leq P\left\{\sup_{\alpha_A} \hat{Q}_n^*(\alpha_A) \geq x\right\} \\ &\Longrightarrow P\left\{\sup_{\alpha_A} Q^*(\alpha_A) \geq x\right\}. \end{aligned} \tag{33}$$

The distribution of $\sup_{\alpha_A} Q^*(\alpha)$ is generally non-standard, but it is completely characterized by its covariance function:

$$K^*(\alpha_1, \alpha_2) = \frac{K(\alpha_1, \alpha_2)}{\sqrt{V(\alpha_1)}\sqrt{V(\alpha_2)}}. \tag{34}$$

The consistent estimators of the components on the right-hand side are given by eq.(23). Therefore, it is possible to obtain the approximate distribution of $\sup_{\alpha_A} Q^*(\alpha_A)$ from the empirical distribution of the random draws, $\sup_{\alpha_A} \hat{Q}_n^*(\alpha_A)$.

2.3 Computing statistics and asymptotic p-values

To compute the standardized LR statistics, we first generate samples under the null hypothesis. Since the model follows independent and identically distributed (i.i.d.) normal distribution, we generate samples of i.i.d. normal observations, using the random generator of the SFMT Mersenne-Twister 19937 (GAUSS software). The sample size is set to the number of observations used for estimation. Then, we compute the maximum likelihood (ML) estimates of the model under the null hypothesis to obtain the estimates of $\mathcal{L}_n^C(\alpha_N)$.

Next, we estimate $\theta(\alpha)$ of the model under the local alternatives for each combination of $\alpha = (\mu_d, p, q)$ using some grids of values. With these estimates, we compute the estimates of $\mathcal{L}_n^C(\alpha_A)$ and thus $LR_n(\alpha_A)$ that is divided by its standard deviation (eq.(28)). Then, we choose the maximum value of $\widehat{LR_n^*}$ (eq.(29)). The set of the grid points is summarized in Table 2 and Table 3. As long as computation results in normal convergence, we choose the range for μ_d so as to cover the ML estimates of the Markov switching model. The ranges of p and q are set as wide as possible.

Turning to asymptotic p-values associated with the $\widehat{LR_n^*}$ statistic in eq.(29), we resort to the Monte Carlo simulation to compute them. Following Hansen (1992, 1996), we use the expression below:

$$\widetilde{LR}_n^*(\alpha)=\frac{\sum_{k=0}^{M}\sum_{t=1}^{n}\hat{q}_t(\alpha)u_{t+k}}{\sqrt{M+1}\sqrt{\hat{V}_n(\alpha)}}, \tag{35}$$

where $u_t(t=1, 2, \cdots, n+M)$ is a random sample of $N(0, 1)$ variables. The process of eq.(35) would approximately give rise to the process of $\hat{Q}_n^*(\alpha)$. Then, we obtain $\sup_{\alpha_A}\hat{Q}_n^*(\alpha_A)$, which is supposed to converge to $\sup_{\alpha_A}Q^*(\alpha_A)$ as n is getting large. We use 1000 Monte Carlo samples to calculate asymptotic p-values associated with the $\widehat{LR}_n^*$ statistic. We set M at the values from 0 to 6 in the next section.

3 Empirical Results

We use the coincident indicators of composite indices, complied by Economic and Social Research Institute (ESRI) affiliated with the Cabinet Office, Government of Japan. ESRI routinely examines and revises the composition of indicators. The latest revision, the 13th revision, was made in March 2021. However, we use the 12th-revision data set because it gives the longest time series at present. The coincident indicators consist of ten series from 1975 to 2020 (see Table 1).

We estimate the Markov switching model not only with the whole sample but with two subsamples: we split the sample periods in February 1991 because Figure 1 and Figure 2 show the indicators have upward trends up to February 1991. This is the peak of the 11th business cycle of Japan after World War II, and around the time that the Japanese asset bubble burst. The dependent variable is the rate of change (%) of each indicator, and its lagged variable appears on the right-hand side of the model equation. Note that both of the retail sales value (C6) and the wholesale sales value (C7) are in the rate of change in the original data set (see Figure 3).

We use 20 grid points for μ_d and 11 points for p and q to compute the standardized likelihood-ratio statistics and p-values. The detailed information is given in footnotes of Table 5 through Table 9 for the whole-sample estimation, and in Table 2 and Table 3 for the subsample cases. The ML estimates, the test statistics, and their associated p-values are provided in Table 5 through Table 19.

Table 4 categorizes the indicators in terms of the range of the estimated asymptotic p-values. The conventional wisdom suggests that the one-state autoregressive model (AR1) be appropriate for the shipment of durable consumer goods (C3), the non-scheduled worked hours (C4), and the shipment of investment goods (C5) with the whole sample. It would also conclude that the two-state Markov switching model is better to explain the industrial production (C1), the retail sales (C6), the operating profit (C8), and the effective job offer rate (C9). As for the shipment of producer goods (C2), the wholesale sales (C7), and the exports (C10), the judgements may vary from researcher to researcher.

We also find that, in Table 4, the subsamples give a distinct picture. When we use the subsample from January 1975 to February 1991, we find that six series out of the ten indicators are congruous with the one-state model and only one series with the two-state model. In contrast, we find that five series are accordant with the two-state model and four with the one-state model when we use the subsample after March 1991. That is, the recent data including 1990s and 2000s are more amenable to the two-state Markov switching model, as found by Carrasco et al. (2014) and Qu and Zhuo (2021).

The retail sales (C6) is compatible with the two-state model with both the whole sample and the subsamples. After March 1991, the two-state model can be used for the industrial production (C1), the shipment of producer goods (C2), and the exports (C10). Since the producer goods consist of raw materials, parts, fuel and the like for production, the observed shipment mainly reflects the production decision. Therefore, all these series are a mirror of economic decision by domestic producers and final users at both ends of commodity trading process. Note that the exports reflect a mixture of purchasing decisions by intermediate distributors and final users as well as the business cycles in foreign countries. As observed in Figure 2, the exports volume show an upward trend until November 2008, just after Lehman Brothers' collapse in September. Such a discrete shock might be better described by the two-state model.

As observed in Table 10 (C1), Table 11 (C2), Table 15 (C6), and Table 19 (C10), the common feature of the estimation results with these series is that the state 1 ($S=1$) is highly persistent with more than a 97% probability, indicating a small but positive average growth rate ranging from 0.16% to 0.46%, and that the state 0 ($S=0$) is less persistent with as high as a 64% probability, indicating a large but negative average growth rate of -13% to -10%. In addition to these series, the operating profit (C8) can also modeled as a two-state Markov switching process, but the estimates in Table 17 indicate a small and negative average growth (-0.31%) in the state 0 with a high persistence probability of 99%, and a large growth rate (57%) in the state 1 with almost zero probability to last. That is, the operating profit jumps up very occasionally, and continues to decrease once it drops.

In contrast, the shipment of durable consumer goods (C3) and the shipment of investment goods (C5) follow a one-state model. The long-run economic decision may not be amenable to the two- state Markov switching model. It is interesting to observe that the p-values are getting smaller with the recent data of the shipment of durable consumer goods, and the other way around with those of the the shipment of investment goods. That is, the two-state model might be getting suited for describing consumers' decision rather than firms' investment decision. Further, the wholesale sales (C7) come from the intermediate transactions in the channel of distribution. Although Table 16 shows that p-values get smaller with the recent data, the null hypothesis of the one-state model is still acceptable. Thus, the intermediate transactions in the distribution seem better modeled by the one-state model.

Finally, we turn to two indicators from the labor market: the non-scheduled worked hours (C4) and the effective job offer rate (C9). Table 13 (C4) and Table 18 (C9) show a substantial decrease in p-values with the recent subsamples. Particularly, we might judge that the two-state model is well suited for the effective job offer rate because the one-state model is decisively rejected with the whole sample (see Table 9). However, we could still argue that the one-state model is good enough based on the estimation results with the subsamples. Since the results are ambiguous, we need more data to judge.

4 Discussion

This paper investigates whether validity of a two-state Markov switching model depends on sample periods. We apply the standardized likelihood-ratio (LR) test proposed by Hansen (1992) to the coincident indicators of Japan at monthly frequency. The main findings are as follows. First, we find that it depends on sample periods whether the one-state or the two-state model is valid. In the recent sample period after 1991, five out of the ten indicators are amenable to the two-state model. The same observation is made by Carrasco et al. (2014) for US real GNP data. Second, the indicators related to production and final users' purchasing decision are amenable to the two-state model in the recent sample period. Third, the one-state model is better suited for the intermediate transactions and the shipments of goods with durability such as investment goods and consumer durables.

A couple of caveats are in order. First, we need more data to assess validity of the two-state model, particularly, for the labor-market data such as non-scheduled worked hours and effective job offer rate. Second, we only consider the two-state model that has the intercept switching between the states. As pointed out by Carrasco et al. (2014), a two-state model with the error-variance switching between the states might show a better fit for the recent data. Finally, we need to work on clarifying the economic theoretical implications of the valid two-state model to derive useful insights for economic analysis. These are subjects for future research.

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Table 1 Coincident Indicators of Japan (the 12th Revision*)

Label	Series Name	Sample Period
C1	Index of Industrial Production (Mining and Manufacturing),	Jan.1975 - Dec. 2020.
C2	Index of Producer's Shipments (Producer Goods for Mining and Manufacturing)	Jan.1975 - Dec. 2020.
C3	Index of Producer's Shipment of Durable Consumer Goods	Jan.1975 - Dec. 2020.
C4	Index of Non-Scheduled Worked Hours (Industries Covered)	Jan.1975 - Dec. 2020.
C5	Index of Producer's Shipment (Investment Goods Excluding Transport Equipment)	Jan.1975 - Dec. 2020.
C6	Retail Sales Value (Change From Previous Year, %)	Jan.1975 - Dec. 2020.
C7	Wholesale Sales Value (Change From Previous Year, %)	Jan.1975 - Dec. 2020.
C8	Operating Profits (All Industries, 100 Mil. Yen)**	Jan.1975 - Sept. 2020.
C9	Effective Job Offer Rate (excl. New School Graduates, Times: # of job offers / # of active job seekers.)	Jan.1975 - Dec. 2020.
C10	Exports Volume Index	Jan.1975 - Dec. 2020.

* Monthly data. Feb. 25th 2021, published by Economic and Social Research Institute, Japan.

** Only quarterly series are available. A linear interpolation is used to obtain monthly series.

† The base year of each index is 2015.

Table 2 Simulation Grid of Parameter Values: μ_d

Data Set Label	Before Feb. 1991			After Mar. 1991		
	Init. Val.	Step Size	#	Init. Val.	Step Size	#
C1	0.10	0.10	20	0.10	0.70	20
C2	0.10	0.10	20	1.00	1.00	20
C3	1.00	0.50	20	1.00	1.70	20
C4	0.10	0.10	20	0.10	0.50	20
C5	-0.10	-0.20	20	-0.10	-0.60	20
C6	0.10	0.15	20	0.10	0.60	20
C7	0.10	0.70	20	0.10	0.70	20
C8	0.10	0.70	20	0.10	0.70	20
C9	0.10	0.50	20	0.10	0.50	20
C10	0.10	0.50	20	0.10	0.70	20

Note: The column "#" indicates the number of grid points.

Table 3　Simulation Grid of Parameter Values: p and q

Data Set Label	Before Feb. 1991			After Mar. 1991		
	Init. Val.	Step Size	#	Init. Val.	Step Size	#
C1	0.005	0.099	11	0.005	0.099	11
C2	0.005	0.099	11	0.092	0.090	11
C3	0.095	0.090	11	0.095	0.090	11
C4	0.095	0.090	11	0.095	0.090	11
C5	0.095	0.090	11	0.095	0.090	11
C6	0.095	0.090	11	0.095	0.090	11
C7	0.095	0.090	11	0.095	0.090	11
C8	0.095	0.090	11	0.005	0.099	11
C9	0.005	0.099	11	0.005	0.099	11
C10	0.005	0.099	11	0.095	0.090	11

Note: The column "#" indicates the number of grid points.

Table 4　p-Value of the Standardized LR statistic

Period	p-value ＜ 1%	1% ≦ p-value ＜ 5%	5% ≦ p-value
Jan.1975-Dec.2020	C1 (Production) C6 (Retail Sales) C8 (Operating Profits) C9 (Job Offer)	C2 (Producers Goods) C7 (Whole Sales) C10 (Exports)	C3 (Durable Goods) C4 (Non-sched. Worked) C5 (Investment Goods)
Jan.1975-Feb.1991	C6 (Retail Sales)	C2 (Producers Goods) C5 (Investment Goods) C9 (Job Offer)	C1 (Production) C3 (Durable Goods) C4 (Non-sched. Worked) C7 (Whole Sales) C8 (Operating Profits) C10 (Exports)
Mar.1991-Dec.2020*	C1 (Production) C2 (Producers Goods) C6 (Retail Sales) C8 (Operating Profits) C10 (Exports)	C9 (Job Offer)	C3 (Durable Goods) C4 (Non-sched. Worked) C5 (Investment Goods) C7 (Whole Sales)

Note: * Sample period ends in Sept. 2020 for C8.

Table 5 Estimates of Markov Switching Model: Data C1 and C2

Model Parameter	C1: Industrial Production	C2: Shipments Prod. Goods
μ	-11.2495	-11.7453
(std. error*)	(1.2817)	(1.0596)
μ_d	11.5238	12.0523
(std. error*)	(1.2861)	(1.0656)
ϕ_1	-0.1807	-0.0191
(std. error*)	(0.0476)	(0.0640)
σ	1.4867	1.5984
(std. error*)	(0.0736)	(0.0854)
p	0.9945	0.9945
(std. error*)	(0.0032)	(0.0032)
q	0.5705	0.5703
(std. error*)	(0.1810)	(0.1694)
Log-Likelihood	-1021.90	-1061.74
# of obs.	550	550
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	5.9671	4.4366
Bandwidth (M)	p-Value	p-Value
0	0.0000	0.0000
1	0.0000	0.0020
2	0.0000	0.0090
3	0.0000	0.0130
4	0.0000	0.0150
5	0.0000	0.0270
6	0.0000	0.0330

Note: The dependent variable is the rate of change of each variable.
*Heteroskedastic consistent estimate of standard error.
Grid of μ_d: 0.1 to 2.0 by step size of 0.1.
Grid of p and q: 0.005 to 0.995 by step size of 0.099.

Table 6　Estimates of Markov Switching Model: Data C3 and C4

Model Parameter	C3: Shipment Durable Goods	C4: Non-Scheduled Worked Hours
μ	-30.7170	-8.0794
(std. error*)	(4.6270)	(1.5677)
μ_d	31.1234	8.1859
(std. error*)	(4.6330)	(1.5750)
ϕ_1	-0.0569	0.0672
(std. error*)	(0.1065)	(0.1236)
σ	3.7609	1.2959
(std. error*)	(0.2782)	(0.0867)
p	0.9946	0.9963
(std. error*)	(0.0032)	(0.0020)
q	0.2518	0.6679
(std. error*)	(0.2178)	(0.1879)
Log-Likelihood	-1529.57	-939.745
# of obs.	550	550
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	3.2952$^{a)}$	3.8985$^{b)}$
Bandwidth (M)	p-Value	p-Value
0	0.0230	0.0010
1	0.0370	0.0130
2	0.0400	0.0300
3	0.0510	0.0350
4	0.0580	0.0430
5	0.0680	0.0430
6	0.0700	0.0500

Note: The dependent variable is the rate of change of each variable.
*Heteroskedastic consistent estimate of standard error.
a) Grid of μ_d: 1 to 10.5 by step size of 0.5.
b) Grid of μ_d: 0.1 to 2.0 by step size of 0.1.
Grid of p and q: 0.095 to 0.995 by step size of 0.090.

Table 7 Estimates of Markov Switching Model: Data C5 and C6

Model Parameter	C5: Shipment Investment Goods	C6: Retail Sales Value
μ	0.3071	0.0830
(std. error*)	(0.0976)	(0.1369)
μ_d	-6.4333	2.6805
(std. error*)	(0.9336)	(0.5683)
ϕ_1	-0.3365	2.6106
(std. error*)	(0.0516)	(0.0649)
σ	2.0900	2.6106
(std. error*)	(0.1208)	(0.1588)
p	0.5526	0.9895
(std. error*)	(0.4742)	(0.0079)
q	0.9846	0.9956
(std. error*)	(0.0146)	(0.0035)
Log-Likelihood	-1227.54	-1318.21
# of obs.	550	550
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	2.9186$^{a)}$	6.0572$^{b)}$
Bandwidth (M)	p-Value	p-Value
0	0.0900	0.0000
1	0.1240	0.0000
2	0.1530	0.0000
3	0.1670	0.0000
4	0.1800	0.0000
5	0.1960	0.0000
6	0.2130	0.0000

Note: The dependent variable is the rate of change of each variable.
*Heteroskedastic consistent estimate of standard error.
a) Grid of μ_d: -0.1 to -11.5 by step size of -0.6.
b) Grid of μ_d: 0.1 to 3.9 by step size of 0.2.
Grid of p and q: 0.095 to 0.995 by step size of 0.090.

Table 8　Estimates of Markov Switching Model: Data C7 and C8

Model Parameter	C7: Whole Sales Goods	C8: Operating Profits
μ (std. error*)	-8.2905 (0.8503)	-18.7193 (5.5291)
μ_d (std. error*)	8.5306 (0.8591)	19.1966 (5.7472)
ϕ_1 (std. error*)	0.9082 (0.0198)	0.3238 (0.2779)
σ (std. error*)	2.7682 (0.1035)	3.5058 (0.8557)
p (std. error*)	0.9886 (0.0052)	0.9963 (0.0023)
q (std. error*)	0.4650 (0.0552)	0.7758 (0.1361)
Log-Likelihood # of obs.	-1373.98 550	-1480.26 547
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	4.2735[a)]	8.3338[b)]
Bandwidth (M)	p-Value	p-Value
0	0.0020	0.0000
1	0.0020	0.0000
2	0.0040	0.0000
3	0.0100	0.0000
4	0.0120	0.0000
5	0.0120	0.0000
6	0.0130	0.0000

Note: The dependent variable is the rate of change of each variable.
*Heteroskedastic consistent estimate of standard error.
a) Grid of μ_d: 0.1 to 13.4 by step size of 0.7.
b) Grid of μ_d: 0.1 to 9.6 by step size of 0.5.
Grid of p and q: 0.095 to 0.995 by step size of 0.090.

Table 9 Estimates of Markov Switching Model: Data C9 and C10

Model Parameter	C9: Effective Job Offer Rate	C10: Exports Volume Index
μ	-6.4906	-13.9505
(std. error*)	(0.7684)	(2.1501)
μ_d	6.6406	14.4761
(std. error*)	(0.7648)	(2.1448)
ϕ_1	0.6551	-0.3030
(std. error*)	(0.0333)	(0.0569)
σ	1.4697	2.5692
(std. error*)	(0.0534)	(0.1106)
p	0.9836	0.9944
(std. error*)	(0.0060)	(0.0031)
q	0.0963	0.6400
(std. error*)	(0.0996)	(0.1813)
Log-Likelihood	-1036.55	-1322.68
# of obs.	550	550
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	5.6590[a)]	4.8414[b)]
Bandwidth (M)	p-Value	p-Value
0	0.0000	0.0000
1	0.0000	0.0000
2	0.0000	0.0000
3	0.0000	0.0040
4	0.0000	0.0070
5	0.0000	0.0070
6	0.0000	0.0100

Note: The dependent variable is the rate of change of each variable.
*Heteroskedastic consistent estimate of standard error.
a) Grid of μ_d: 0.1 to 3.9 by step size of 0.2.
b) Grid of μ_d: 0.1 to 9.6 by step size of 0.5.
Grid of p and q: 0.005 to 0.995 by step size of 0.099.

Table 10 Estimates of Markov Switching Model: Data C1

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	-0.8419	-11.0270
(std. error*)	(0.3935)	(1.3445)
μ_d	1.6756	11.1957
(std. error*)	(0.3838)	(1.3445)
ϕ_1	-0.3903	-0.1422
(std. error*)	(0.0639)	(0.0545)
σ	0.9969	1.6076
(std. error*)	(0.0820)	(0.1020)
p	0.7971	0.9914
(std. error*)	(0.0888)	(0.0049)
q	0.0000	0.5703
(std. error*)	(0.0012)	(0.1690)
Log-Likelihood	-302.092	-696.184
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	2.4258	5.6647
Bandwidth (M)	p-Value	p-Value
0	0.2770	0.0000
1	0.2430	0.0000
2	0.2460	0.0000
3	0.2640	0.0000
4	0.2420	0.0000
5	0.2410	0.0002
6	0.2470	0.0003

Note: The dependent variable is the rate of change of Index of Industrial Production (%).
*Heteroskedastic consistent estimate of standard error.

Table 11 Estimates of Markov Switching Model: Data C2

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ (std. error*)	-0.1509 (0.2041)	-11.4766 (1.1341)
μ_d (std. error*)	1.1321 (0.1943)	11.7339 (1.1366)
ϕ_1 (std. error*)	-0.3464 (0.0807)	0.0168 (0.0697)
σ (std. error*)	1.1383 (0.0714)	1.7461 (0.1160)
p (std. error*)	0.9427 (0.0228)	0.9914 (0.0049)
q (std. error*)	0.9285 (0.0367)	0.5697 (0.1943)
Log-Likelihood	-310.870	-725.536
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	3.9724	4.9046
Bandwidth (M)	p-Value	p-Value
0	0.0030	0.0000
1	0.0070	0.0000
2	0.0120	0.0000
3	0.0110	0.0010
4	0.0150	0.0040
5	0.0110	0.0070
6	0.0160	0.0060

Note: The dependent variable is the rate of change of Index of Producer's Shipments of Producer Goods (%).
*Heteroskedastic consistent estimate of standard error.

Table 12 Estimates of Markov Switching Model: Data C3

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ (std. error*)	0.7383 (0.1670)	-31.8767 (21.5559)
μ_d (std. error*)	5.4584 (1.0658)	32.1258 (21.3019)
ϕ_1 (std. error*)	-0.4651 (0.0660)	0.0000 (0.1743)
σ (std. error*)	2.1557 (0.1293)	4.3204 (0.5330)
p (std. error*)	0.2278 (0.2192)	0.9932 (0.0142)
q (std. error*)	0.9787 (0.0149)	0.2927 (0.4324)
Log-Likelihood	-432.880	-1041.91
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	1.2725	4.9046
Bandwidth (M)	p-Value	p-Value
0	0.8970	0.0470
1	0.9120	0.0730
2	0.8960	0.0880
3	0.8710	0.0820
4	0.8670	0.0970
5	0.8450	0.0910
6	0.8460	0.1160

Note: The dependent variable is the rate of change of Index of Producer's Shipment of Durable Consumer Goods (%).
*Heteroskedastic consistent estimate of standard error.

Table 13 Estimates of Markov Switching Model: Data C4

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	-0.0909	-7.9405
(std. error*)	(0.1011)	(1.6690)
μ_d	1.0552	7.9982
(std. error*)	(0.1745)	(1.6771)
ϕ_1	-0.2031	0.0892
(std. error*)	(0.0939)	(0.1587)
σ	0.8705	1.4302
(std. error*)	(0.0525)	(0.12154)
p	0.8946	0.9943
(std. error*)	(0.0558)	(0.0029)
q	0.9521	0.6705
(std. error*)	(0.0268)	(0.1938)
Log-Likelihood	-263.672	-648.266
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	2.5137	3.6265
Bandwidth (M)	p-Value	p-Value
0	0.2260	0.0100
1	0.2050	0.0140
2	0.2150	0.0320
3	0.2020	0.0380
4	0.2160	0.0420
5	0.2010	0.0570
6	0.1970	0.0580

Note: The dependent variable is the rate of change of Index of Non-Scheduled Worked Hours (%).
*Heteroskedastic consistent estimate of standard error.

Table 14　Estimates of Markov Switching Model: Data C5

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	3.1099	0.1428
(std. error*)	(0.6458)	(0.1245)
μ_d	-2.8846	-6.4072
(std. error*)	(0.6270)	(0.6882)
ϕ_1	-0.3462	-0.3290
(std. error*)	(0.06447)	(0.0595)
σ	1.5450	2.2572
(std. error*)	(0.1291)	(0.1284)
p	0.8736	0.5749
(std. error*)	(0.06904)	(0.1903)
q	0.0000	0.9797
(std. error*)	(0.0012)	(0.0119)
Log-Likelihood	-382.534	-828.271
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	3.1196	2.4863
Bandwidth (M)	p-Value	p-Value
0	0.0440	0.2120
1	0.0410	0.2540
2	0.0320	0.2820
3	0.0320	0.3350
4	0.0350	0.3760
5	0.0260	0.3790
6	0.0270	0.4030

Note: The dependent variable is the rate of change of Index of Producer's Shipment of Investment Goods (%).
*Heteroskedastic consistent estimate of standard error.

Table 15 Estimates of Markov Switching Model: Data C6

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	-0.1842	-9.9003
(std. error*)	(0.4469)	(8.0887)
μ_d	2.5606	10.0786
(std. error*)	(0.7697)	(8.1353)
ϕ_1	0.8588	0.6023
(std. error*)	(0.0493)	(0.0887)
σ	2.2764	2.1047
(std. error*)	(0.2252)	(0.2402)
p	0.0703	0.9772
(std. error*)	(0.2420)	(0.0094)
q	0.3078	0.0269
(std. error*)	(0.5173)	(1.7057)
Log-Likelihood	-451.721	-807.530
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	4.4105	4.0296
Bandwidth (M)	p-Value	p-Value
0	0.0002	0.0010
1	0.0001	0.0020
2	0.0000	0.0040
3	0.0000	0.0040
4	0.0000	0.0030
5	0.0010	0.0060
6	0.0010	0.0040

Note: The dependent variable is the rate of change of Retail Sales Value (%).
*Heteroskedastic consistent estimate of standard error.

Table 16　Estimates of Markov Switching Model: Data C7

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	0.6839	-8.4396
(std. error*)	(0.2956)	(0.8766)
μ_d	7.0256	8.1619
(std. error*)	(1.1725)	(0.8550)
ϕ_1	0.8495	0.7556
(std. error*)	(0.0395)	(0.0326)
σ	2.6613	2.5745
(std. error*)	(0.2234)	(0.1044)
p	0.2647	0.9896
(std. error*)	(0.2555)	(0.0068)
q	0.9576	0.7625
(std. error*)	(0.0237)	(0.1149)
Log-Likelihood	-484.472	-865.629
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	2.7227	3.5582
Bandwidth (M)	p-Value	p-Value
0	0.1370	0.0260
1	0.1360	0.0330
2	0.1230	0.0510
3	0.1320	0.0720
4	0.1410	0.0700
5	0.1270	0.0730
6	0.1410	0.0930

Note: The dependent variable is the rate of change of Wholesale Sales Value (%).
*Heteroskedastic consistent estimate of standard error.

Table 17 Estimates of Markov Switching Model: Data C8

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Sept. 2020 Estimate
μ	0.1338	-0.3058
(std. error*)	(0.1015)	(0.1341)
μ_d	11.5123	57.3896
(std. error*)	(0.7354)	(18.2781)
ϕ_1	0.7647	0.8677
(std. error*)	(0.0842)	(0.0873)
σ	1.3069	3.1013
(std. error*)	(0.2371)	(0.4673)
p	0.0004	0.0007
(std. error*)	(0.0004)	(0.005)
q	0.9948	0.9943
(std. error*)	(0.0052)	(0.0040)
Log-Likelihood	-330.078	-912.761
# of obs.	192	353
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	1.9519	11.6096
Bandwidth (M)	p-Value	p-Value
0	0.4140	0.0000
1	0.4130	0.0000
2	0.4000	0.0000
3	0.3950	0.0000
4	0.3830	0.0000
5	0.3680	0.0000
6	0.3760	0.0000

Note: The dependent variable is the rate of change of Wholesale Sales Value (%).
*Heteroskedastic consistent estimate of standard error.

Table 18　Estimates of Markov Switching Model: Data C9

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	-8.6302	-4.8030
(std. error*)	(0.3701)	(0.4367)
μ_d	8.9615	4.8873
(std. error*)	(0.3824)	(0.4445)
ϕ_1	0.5363	0.7167
(std. error*)	(0.0673)	(0.0345)
σ	1.6590	1.3016
(std. error*)	(0.0953)	(0.0509)
p	0.9838	0.9816
(std. error*)	(0.0093)	(0.0080)
q	0.0003	0.1442
(std. error*)	(0.0002)	(0.1431)
Log-Likelihood	-385.064	-629.070
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	3.7152	3.9848
Bandwidth (M)	p-Value	p-Value
0	0.0120	0.0006
1	0.0120	0.0002
2	0.0140	0.0002
3	0.0150	0.0100
4	0.0230	0.0100
5	0.0250	0.0130
6	0.0300	0.0120

Note: The dependent variable is the rate of change of Effective Job Offer Rate (Excluding New School Graduates) (%).
*Heteroskedastic consistent estimate of standard error.

Table 19 Estimates of Markov Switching Model: Data C10

Model Parameter	Jan. 1975 - Feb.1991 Estimate	Mar. 1991 - Dec. 2020 Estimate
μ	0.4541	-13.1026
(std. error*)	(0.1788)	(2.1634)
μ_d	7.3860	13.5634
(std. error*)	(0.8112)	(2.1467)
ϕ_1	-0.4271	-0.2238
(std. error*)	(0.1019)	(0.0819)
σ	2.3821	2.4473
(std. error*)	(0.1605)	(0.1352)
p	0.0000	0.9912
(std. error*)	(0.0000)	(0.0051)
q	0.9659	0.6380
(std. error*)	(0.0181)	(0.1628)
Log-Likelihood	-459.581	-845.471
# of obs.	192	356
	Testing Hypo.: One State (Null) vs. Two States (Alt.)	
Std. LR stat.	2.1101	5.5211
Bandwidth (M)	p-Value	p-Value
0	0.4930	0.0000
1	0.4450	0.0000
2	0.4190	0.0000
3	0.3940	0.0000
4	0.3860	0.0010
5	0.3680	0.0010
6	0.3850	0.0020

Note: The dependent variable is the rate of change of Exports Volume Index (%).
*Heteroskedastic consistent estimate of standard error.

Figure 1　Coincident Indicators : C1, C2, C3 and C5

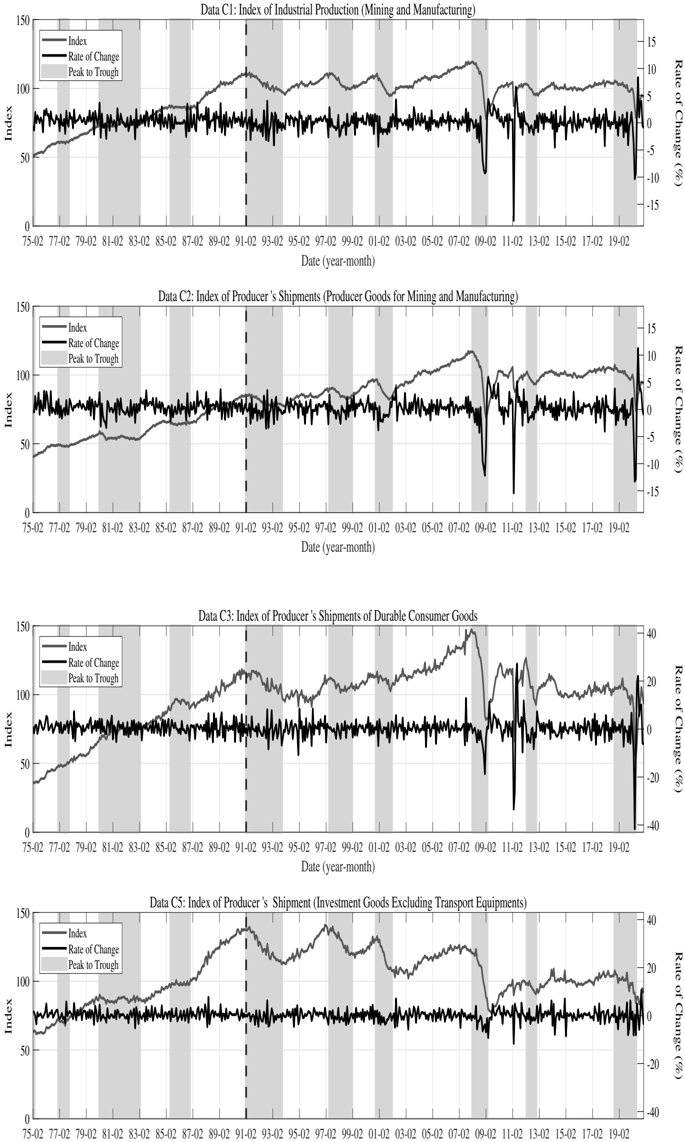

Figure 2 Coincident Indicators : C4, C10, C8 and C9

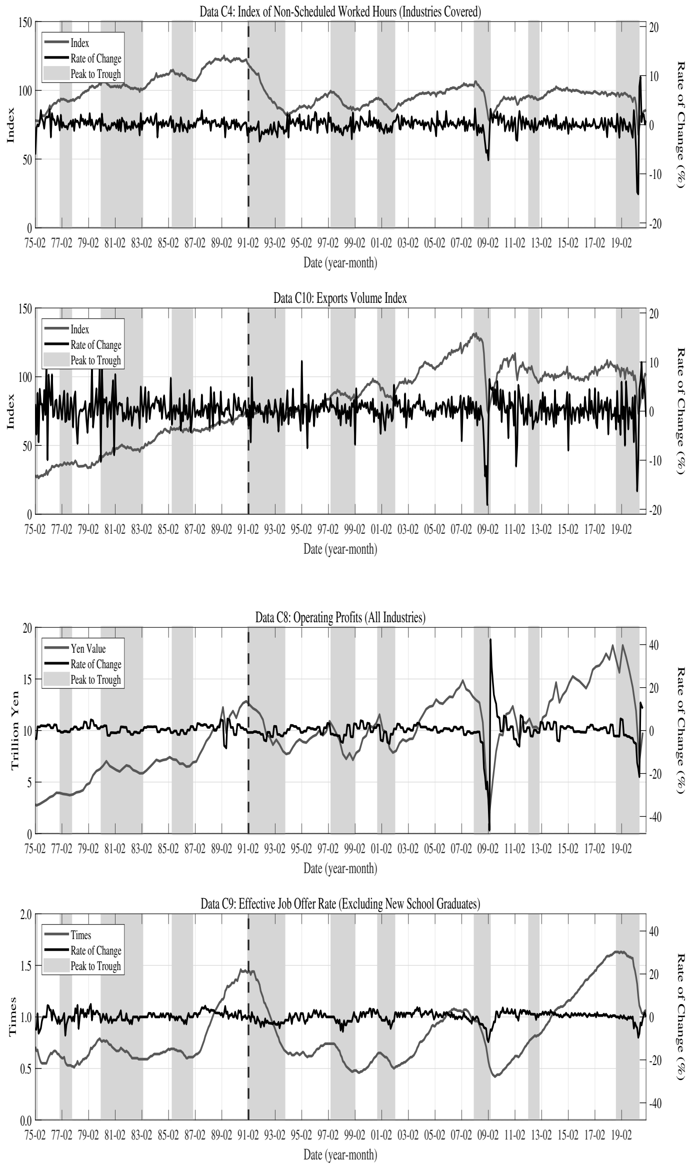

Figure 3　Coincident Indicators : C6 and C7