Coincident Index and Reference Cycle

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Abstract

This paper seeks an alternative method to make the coincident composite index that is simpler than the existing method and is consistent with the official reference cycle as much as possible. We first examine how accurately the coincident composite index trace the reference cycle, using the business cycle data of Japan. Then, we investigate aggregation methods to construct the composite index from individual indicators in terms of normalization of each indicator, smoothing methods, and dating algorithm. The main findings are as follows. First, use of an outlier trimming procedure and interquartile ranges plays only a minor role in dating peaks and troughs. Secondly, when filtering methods are used to seasonally adjust series, the Bry-Boschan algorithm can be simplified to a great extent and does not need the Spencer smoothing, the 12-month moving averages, and the trimming procedure. Thirdly, the Butterworth filter has the same effects on dating results as the interquartile ranges. Thus, the smoothing via filtering reduces variation good enough.

Key words: composite index, coincident indicators; reference dates

JEL classification: E32

1 Introduction

Business cycle has been one of the main macroeconomic subjects in the academic literature as well as in practice since Burns and Mitchell (1946). Theoretical developments lead to the real business cycle theory (Kydland and

Prescott, 1982; Prescott, 1986) and the New Keynsian framework (Clarida, Galí, and Gartler, 1999, 2000), and spawned various models (see Woodford, 2003; Walsh, 2010). On the empirical front, a long-lasting concern is to understand the business-cyclical characteristics: identification of the reference dates of turning points, duration and amplitude of the cycles, and synchronization among regions and industries. Recent developments and subjects are concisely described in Harding and Pagan (2016).

There are two types of indices of the business cycles: diffusion and composite indices. Further, we have two types of diffusion indices in Japan: *current* and *historical* indices. Each index consists of leading, coincident, and lagging indicators. The number of indicators to create composite indices are different agent by agent around the world. For example, Economic and Social Research Institute (ESRI) in Japan currently adopts 9 series to calculate the coincident index, while 4 series are selected at the Conference Board in the United States (the Conference Board, 2001, p.49).

The diffusion indices are often used to determine turning points of the business cycle, that is, extract the reference cycle (Harding and Pagan, 2016, p.52). The diffusion indices are typically compiled as follows. First, we determine the states of expansion and contraction. For example, we might look at the changes of each series for the diffusion indices. Then, we count the number of positive (or negative) changes at each point of time, and divide it by the total number of the series adopted as an indicator. If more than 50% of the series indicate positive (or negative) changes, the economic state is considered in an expansion (or contraction, respectively). Thus, the trough (or peak) can be found at a time, say, t, right before the share of the positive (negative) changes has just gone beyond 50% at time $t+1$.

ESRI computes the changes from three months ago for each indicator to compute the current diffusion indices. It gives 1 for a positive change, 0.5 for no change, and 0 for a negative one, sums up these numbers all over the series, divides them by the number of the series and multiply by 100. Thus, the diffusion index takes 100 at a point of time when all the series indicate positive changes, 50 in case of no changes at all and 0 in case of negative changes only. On the other hand, ESRI uses the Bry-Boschan (BB) algorithm to make the historical indices. It applies the BB algorithm to each series to find turning points. Then, it presupposes positive changes during the period of a trough to a peak and negative from a peak to a trough for each series, and obtains the historical index by computing the ratio of the number of the series with positive changes to the total number of the series, multiplied by 100. Then, an economy is supposed at a peak when a coincident historical index goes down to less than 50% in the next period, and at a trough when it goes up to more than 50%.

While the diffusion indices play an important role in determination of the turning points, the composite indices could be appropriate for analyses of the amplitude of the cycles, rate of change in the cycles, and forecasting. Specifically, the leading indicators attract a great attention because it is presumed appropriate for forecasting economic states in the future (Lahiri and Moore, 1991). However, the coincident and the lagging indices are not much used in academic research, and the composite indices derived from them are far less employed. Stock and Watson (1991) estimated the so-called single-index model for the U.S. with four coincident indicators and compare with the composite coincident index. Harding and Pagan (2006) used them to investigate whether their proposed algorithm could replicate the NBER (National Bureau of Economic Research) reference cycle.

On the one hand, the lagging composite index rarely draws attention in the literature, and on the other, the coincident composite index should be useful to understand present states of economies. Since the latter is composed of the coincident indicators, it is also expected to indicate the same peaks and troughs as the reference cycle. If the coincident composite index accurately follows the reference cycle, we can use it to understand the amplitude of the business cycle and the magnitude of plunges or booms. Further, it may give a criterion of validity of the business-cycle models and the related econometric models. But, this is not the case in the literature. For example. Canova (1994, 1998, 1999) investigated various detrending methods with many kinds of quarterly data, adopting the reference cycle as a criterion instead of the composite index. Since the coincident composite index is complied in a different way from the reference cycle in that the former is the average of the coincident indicators and the latter is based on the counts of signs of their changes and other information available, we need to investigate how closely the index follows the reference cycle.

This paper attempts to find an alternative method to make the coincident composite indices that is simpler than the existing method and produces a dating result consistent with the official reference cycle as much as possible. We start empirical analyses by examining how accurately the coincident composite indices trace the reference cycle, using the business cycle data of Japan. Then, we investigate aggregation methods to compute the composite index from the individual indicators in terms of normalization of each indicator, smoothing methods, and dating algorithm.

The main findings are as follows. First, although ESRI uses an outlier trimming procedure and interquartile ranges for a scaling measure, these devices play only a minor role in dating peaks and troughs. Secondly, when filtering methods are used to seasonally adjust series, the Bry-Boschan algorithm can be simplified to a great extent and does not need the Spencer smoothing, the 12-month moving averages, and a trimming procedure.Thirdly, the Butterworth filter has the same effects on dating results as the interquartile ranges. Therefore, it reduces variation of the series good enough.

The rest of the paper is organized as follows. In section 2, we summarize the aggregation method of the ESRI to make the composite coincident index. Section 3 explains the algorithm of the Bry-Boschan procedure to date peaks and troughs. It is followed by a brief review of filtering methods used in the paper in section 4. In section 5, we analyze the coincident indices of Japan. The final section is allocated to discussion.

2 Indicator Aggregation: ESRI Method

In this section, we explain how ESRI (Economics and Social Research Institute) computes the composite indices: the leading, the coincident and the lagging index (Economic and Social Research Institute, 2015). Its method consists of outlier removal and aggregation, and is common to the three indices. Let $x_i^j(t)$, where j indicates a type of composite indices ($j=L$: leading, C: coincident, Lag: lagging), i a series used in each index, and t a point of time. Then, we first calculate a symmetric rate of change as follows:

Step I: Symmetric Rate of Change

$$r_i^j(t)=200\times\frac{x_i^j(t)-x_i^j(t-1)}{x_i^j(t)+x_i^j(t-1)} \tag{1}$$

Step II: Outlier Removal

Next, outliers are removed. Let $Q3_i^j-Q1_i^j$ an interquartile range of $r_i^j(t)$, and outlier-adjusted rate of change $\psi_i^j(t)$

$$\phi_i^j(t)=\begin{cases} -k(Q3_i^j-Q1_i^j), & \dfrac{r_i^j(t)}{Q3_i^j-Q1_i^j}<-k \\ r_i^j(t), & -k\leqq\dfrac{r_i^j(t)}{Q3_i^j-Q1_i^j}\leqq k \\ k(Q3_i^j-Q1_i^j), & k<\dfrac{r_i^j(t)}{Q3_i^j-Q1_i^j} \end{cases} \tag{2}$$

Here, k is a constant threshold value that is set so as to trim 5% at edges of $r_i^j(t)$ $(j=C)$, where t ranges from January 1980 to the latest December. k takes a value of 2.02 as of November 2011. This rule is used to remove outliers in the computational procedure up until August 2011. A similar rule is used after September 2011, which is explained later.

Step III: Trend

Two types of averages are used. The first one is called 'trend of individual series' by ESRI. It is an avereged outlier-free rate of change of each series in time domain, computed as follows:

$$\mu_i^j(t)=\frac{1}{60-s}\sum_{\tau=t-59}^{t-s}\phi_i^j(\tau) \tag{3}$$

where s is the number of missing values. That is, it is an averaged value over the last 60 months. The second average is computed across series of the coincident index:

$$\bar{\mu}_i^C(t)=\frac{1}{n^C}\times\sum_{i=1}^{n^C}\mu_i^C(t) \tag{4}$$

where n^C is the number of series used to compute the coincident index.

Step IV: Standardized Rate of Change

The rate of change is standardized with the interquartile range for each series:

$$z_i^j(t)=\frac{\psi_i^j(t)-\mu_i^j(t)}{Q3_i^j-Q1_i^j} \tag{5}$$

The average rate of change is computed except missing values of $z_i^j(t)$ as follows:

$$\overline{Z}^j(t)=\frac{1}{n^j-s^j(t)}\times\sum_{i=1}^{n^j}z_i^j(t) \tag{6}$$

where $s^j(t)$ is the number of series that have missing values at time t.

Step V: Synthesis

The overall average rate of change is computed as follows:

$$V^j(t)=\overline{\mu}_i^C(t)+\overline{Q3-Q1}^j\times\overline{Z}^j(t) \tag{7}$$

where

$$\overline{Q3-Q1}^j=\frac{1}{n^j}\times\sum_{i=1}^{n^j}(Q3_i^j-Q1_i^j) \tag{8}$$

Step VI: Composite Index

To compute a composite index, the following indexation is used:

$$I^j(t)=I^j(t-1)\times\frac{200+V^j(t)}{200-V^j(t)} \tag{9}$$

where the initial value of $I^j(t)$ is set to 1. Then, the composite index is obtained as follows:

$$CI^j(t)=\frac{I^j(t)}{I^j}\times 100 \tag{10}$$

where I^j is an average in the base year.

Since September 2011, a refinement has been made in *Step II*. The basic idea is to make outlier adjustments only to series-specific parts of the standardized rate of change, so that a shock common to all the series should be excluded from outlier removal. First, a time trend for each series is calculated as

$$m_i^j(t) = \frac{1}{60-s} \sum_{\tau=t-59}^{t-s} r_i^j(\tau) \tag{11}$$

where s is the number of series that have missing values at time t. Then, the rate of change is standardized with the interquartile range:

$$\eta_i^j(t) = \frac{r_i^j(t) - m_i^j(t)}{Q3_i^j - Q1_i^j} \tag{12}$$

Let the median of $\eta_i^j(t)$ across series denoted by $\tilde{\eta}^j(t)$. Now, subtracting $\tilde{\eta}^j(t)$ from both sides of eq. (12) and rearranging it, we obtain:

$$\begin{aligned} r_i^j(t) &= \underbrace{(\eta_i^j(t) - \tilde{\eta}^j(t))(Q3_i^j - Q1_i^j) + m_i^j(t)}_{\text{specific parts}} + \underbrace{\tilde{\eta}^j(t)(Q3_i^j - Q1_i^j)}_{\text{common part}} \\ &= \hat{r}_i^j + \tilde{r}^j \end{aligned} \tag{13}$$

Then, the outlier adjustment is applied to $\hat{r}_i^j$ as follows:

$$\hat{\phi}_i^j(t) = \begin{cases} -\hat{k}(\widehat{Q3}_i^j - \widehat{Q1}_i^j), & \dfrac{\hat{r}_i^j(t)}{\widehat{Q3}_i^j - \widehat{Q1}_i^j} < -\hat{k} \\ \hat{r}_i^j(t), & -\hat{k} \leqq \dfrac{\hat{r}_i^j(t)}{\widehat{Q3}_i^j - \widehat{Q1}_i^j} \leqq \hat{k} \\ \hat{k}(\widehat{Q3}_i^j - \widehat{Q1}_i^j), & \hat{k} < \dfrac{\hat{r}_i^j(t)}{\widehat{Q3}_i^j - \widehat{Q1}_i^j} \end{cases} \tag{14}$$

where $\widehat{Q3}_i^j - \widehat{Q1}_i^j$ an interquartile range of $\hat{r}_i^j(t)$. Further, $\hat{k}$ is a constant threshold value as before that is supposed to trim 5% at edges of $\hat{r}_i^j(t)$ $(j=C)$, where t ranges from January 1985 to the latest December. $\hat{k}$ takes a value of 2.04 as of December 2015. Then, the outlier-free rate of change, $\phi_i^j(t)$ in eq. (2), is obtained as follows:

$$\phi_i^j(t) = \hat{\phi}_i^j + \tilde{r}^j \tag{15}$$

The rest of the procedure follows as already explained.

3 Bry-Boshcan Procedure

The BB procedure is summarized in Table 1. Watson (1994) found some discrepancies between the original description by Bry and Boschan (1971) and the Fortran program they coded. The description here is modified to be consistent with the Fortran codes. The procedure presumes to use seasonally adjusted series. In Step I, outliers, if any, are replaced by the values of the Spencer curve. Here, the outliers are defined as values whose ratios to (or differences in absolute values from, depending on data) the 15-point Spencer curve are larger than 3. 5 standard deviations, a threshold value chosen arbitrarily. This Spencer curve is computed as the 15-month symmetric moving average with particular weights (see Kendall and Stuart, 1966, p.458).

Step II starts with the 12-month moving average (MA12, hereafter) of the outlier-free series. The MA12 is chosen on the ground that the Spencer curve contains too many minor fluctuations. Any date with the highest value among the 6 preceding and the 6 following months is tentatively regarded as the date of a peak. Similarly, any date with the lowest among the 6 preceding and the 6 following months is considered the date of a tentative trough. These peaks and troughs are checked for alternation. For contiguous peaks or troughs, the highest value is chosen for a peak, and the lowest for a trough. If the values are same, we set an earlier date for a peak, and a later date for a trough, respectively. Note that the MA12 filter is not symmetric: 6 lags and 5 leads. At the ends of the sample, it is an one-sided filter. Therefore, phase shifts are introduced, which might cause misinterpretation of timing of economic events.

In Step III, the Spencer curve of the outlier-free series is used to ensure peaks and troughs within ± 6 months, because its turns are heuristically closer to those of the original series than those of MA12. If there are ties within ±6 data points on the Spencer curve, an earlier date is chosen for a peak, and a later date for a trough. After alternation check as in Step II, the duration of a peak to peak or a trough to trough (a full cycle) is enforced to be at least 15 months. If the duration is too short, the lower of two peaks or the higher of two troughs are eliminated. If the values are same, we set an earlier date for a peak, and a later date for a trough, respectively. Alternation check is conducted if any modification.

In Step IV, a further refinement is conducted with a short-term moving average, which is called MCD (Months for Cyclical Dominance) curve. The MCD is obtained as follows. First, we compute the Spencer curve of the original series, taking it as the trend-cycle component. The difference between the original series and the trend-cycle component gives the irregular component. Next, we take the ratio of the average change in the irregular component to that in the trend-cycle component. The change is computed either by the rate of change or by the difference of each component over various time spans. The MCD is the minimum number of months that gives the ratio less than 1. That is, the MCD is the shortest months that it takes for the change in the trend-cycle component to dominate that in the irregular component. The BB procedure confines the MCD between 3 and 6 months. Then, a short-term moving average is computed over the span of MCD, and used to ensure peaks and troughs within ±6 months as in Step III. Alternation is checked as in Step II if modified.

In the final step ('V'), a series of tests are conducted to determine final turns. First, the original series is used to ensure peaks and troughs within ±4 months or ±MCD, whichever is longer (denoted by 'V. 1'). The second test ('V. 2') is alternation check as in Step II. Third ('V.3'), any turn within less than 6 months from the ends is removed. In the fourth test ('V.4'), if the first or the last peak (or trough) takes a value smaller (or greater) than any value between it and the end of the original series, it is removed. In the program used by Watson (1994), the first and the last turns are only compared with the initial and the last data points, respectively, not with all the values between them. Although this could make a nontrivial difference, it does not change the results of the paper. Here, we follow Watson (1994).

The fifth test ('V.5') is to check if the duration of a full cycle is at least 15-month length, as in Step III. The final test ('V.6') is to check whether a phase (peak to trough or trough to peak) duration is at least 5 months. If it is less than 5 months, the two turning points are eliminated. If the violation is found at the last turning point, only the last point is removed. In later experiments, we implement the procedure with several steps skipped to see their effects. We also replace the 12-month-moving-averaged series with the series smoothed by filtering methods to examine the importance of smoothness.

4 Seasonal Adjustment via Bandpass filters

To examine effects of seasonal adjustment on determination of the reference cycle, we use three bandpass filters: Christiano-Fitzgerald filter, Hamming filter, and Butterworth filters. Since the bandpass filters are supposed to extract certain cycles of a signal, it can also extract cycles longer than seasonal cycles. Here, the difference between the bandpass filtering and the conventional seasonal adjustment procedures like X12-ARIMA is whether the cycles shorter than the seasonal cycles are removed or left. The conventional method attempts to remove the seasonal cycles only, while the bandpass filtering removes all the cyclical components shorter than and equal to the seasonal cycles. Because our study concerns business cycles which are supposed to be longer than the seasonal cycles, we have no good reason to leave the shorter cycles in the series. Further, removal of the shorter cycles could give rise to denoising effects so that arbitrary outlier removal is less likely to play a great role in dating business cycles.

Before reviewing filtering methods, we first note several criteria to assess relative performance among those methods in terms of economic analyses. One criterion is whether a method can extract cyclical components to replicate official reference dates of the business cycles. Here, the cyclical components obtained by filtering are considered to be the growth cycle that is supposed to have a close relation to the business cycle. Canova (1994) examined performance of 11 different detrending methods to replicate NBER dating, assuming that the detrending removes a secular component. Similar analyses are conducted by Canova (1999) with 12 methods including Hamilton (1989)' s procedure. They found that the Hodrick-Prescott (HP) filter proposed by Hodrick and Prescott (1997) and a frequency domain filter as an approximation to the Butterworth filter (see Canova, 1998, p. 483) would be the most reliable tools to reproduce the NBER dates. Recently, Otsu (2013) conducted a comparative analysis among band pass filters such as the Christiano-Fitzgerald (CF) filter (Christiano and Fitzgerald, 2003), the Hamming-windowed filter (Iacobucci and Noullez, 2005) and the Butterworth filters (e.g. Gomez, 2001; Pollock, 2000), using Japanese real GDP data. It shows that the Butterworth filters give the business-cycle dates closest to the official reference dates.

Another criterion is *phase shift*. That is to say, detrending or transformation should cause no phase shifts so that it would not change time alignment of events. In general, use of one-sided filters or statistical models with lagged variables alone would cause phase shifts, which may lead to misinterpretation of economic events. Free from phase shifts are two-sided and symmetrical filters such as the Baxter-King (BK) filters (Baxter and King, 1999), the Hamming-windowed filter, and two-sided Butterworth filters. Since a large phase shift tends to lead to a large deviation of estimated business-cycle dates from the official ones, this criterion is closely related to the first criterion.

The third criterion is stability of the estimated components, so that they would not change when more observations become available. Then, filtering procedures had better not be subject to the whole sample. Since most of the procedures involve estimation of coefficients, time-varying weights, or the Fourier transform, their resulting components would be susceptible to data updating. Therefore, it is a matter of degree. Otsu (2011b) examined stability of two types of frequency-domain filtering methods, the Hamming-windowed filter and the Butterworth filters, and one time-varying filtering method in time domain, the CF filtering. It found that the larger the sample size, the more stable the estimated components based on the frequency filtering, and that the sample size of 100 for quarterly data would be good enough to obtain stable estimates in practice. It also showed that the Butterworth filters give the most stable estimates among others. Thus, they might be useful in practice.

The fourth criterion is how much a weight of each cyclical component alters by detrending or transformation, which is called *exacerbation* in Baxter and King (1999). When we use finite time-domain filters to approximate the ideal filter, certain components tend to be magnified or reduced as a result of filtering. To inspect this point, it is useful to look at the frequency response function of the time-domain filter. Then, it would show oscillations over the frequencies of the pass band and the stop band, indicating magnification and reduction of certain components. As the filter length gets longer, the oscillations become more rapid but do not diminish in amplitude. They converge to the band edges or the discontinuity points of the ideal filter, which is called *Gibbs phenomenon*. This phenomenon is attributed to approximation of infinite sum by truncation. This implies that cutting out a part of the Fourier-transformed series discontinuously as in Canova (1998, p. 483) would create the same artificial oscillatory behavior in the estimated components. In light of this criterion, the Butterworth filters and the Hamming-windowed filter have a desirable property because they have flat frequency response functions over the ranges of the pass band and the stop band.

The final criterion is the degree of *leakage* and *compression* as discussed in Baxter and King (1999). That is, detrending or filtering might admit substantial components from the range of frequencies that are supposed to suppress (*leakage*), and lose substantial components over the range to be retained (*compression*). Since these effects depend on the width of transition bands between the pass and the stop bands, it is better to have narrow transition bands. Otsu (2009, 2010) show that the Butterworth filters are least afflicted with leakage and compression effects among others. In the related study, Otsu (2007) examined discrepancies between the ideal filter and several approximate filters, and found that the Butterworth filters give a better approximation than other bandpass filters. This also implies that the Butterworth filters could give rise to the least leakage, compression, and exacerbation effects.

Now we review properties of three methods used later: Christiano-Fitzgerald filter, Hamming filter, and Butterworth filters. To begin with, we consider the following orthogonal decomposition of the observed series x_t:

$$x_t = y_t + \tilde{x}_t \tag{16}$$

where y_t is a signal whose frequencies belong to the interval $\{[-b, -a] \cup [a, b]\} \in [-\pi, \pi]$, while $\tilde{x}_t$ has the complementary frequencies. Suppose that we wish to extract the signal y_t. The Wiener-Kolmogorov theory of signal extraction, as expounded by Whittle (1983, Chapter 3 and 6), indicates y_t can be written as:

$$y_t = B(L)x_t \tag{17}$$

$$B(L)=\sum_{j=-\infty}^{\infty} B_j L^j, \quad L^k x_t \equiv x_{t-k} \tag{18}$$

In polar form, we have

$$B(e^{-iw})=\begin{cases} 1, \text{ for } \omega \in [-b, -a] \cup [a, b] \\ 0, \text{ otherwise} \end{cases} \tag{19}$$

where $0 \leqq a \leqq b \leqq \pi$. Theoretically, we need an infinite number of observations, x_t's, to compute y_t. In practice, the filtering methods approximate y_t by $\hat{y}_t$, a filtered series with a finite filter. To estimate y_t by $\hat{y}_t$, the Christiano-Fitzgerald filtering is performed in the time domain with truncation at both ends of the sample, while other filtering methods in the frequency under the circularity assumption. In application to seasonal adjustment, when we set a to zero and b to the seasonal frequencies concerned, we have power spectra identical to those of the seasonally adjusted series published officially (see Otsu, 2009, p. 212 and p. 219). In the later analysis, we set b to $\frac{\pi}{6}$. Now, we briefly review three filtering methods mentioned above.

4.1 Christiano-Fitzgerald Filter

Christiano and Fitzgerald (2003) seeked an optimal linear approximation with finite sample observations. They solved a minimization problem based on the mean square error (MSE) criterion in the frequency domain: minimization of a weighted sum of differences between the ideal bandpass-filter's weights and their approximates, using a spectral density of observations as a weight. They derived optimal filter weights, assuming a difference-stationary process of observed data with a trend or a drift removed if any.

In their empirical investigations, they examined the effects of the time-varying weights, the asymmetry, and the assumption on the stochastic process.

They compared variance ratios and correlations between the components extracted by the CF filters and the theoretical components based on the data generating process of observations. To evaluate the second moments of the theoretical components, they used the Riemann sum in the frequency domain. They found that the time-varying weights and the asymmetry of the filter contribute to a better approximation, pointing out that the time-varying feature is relatively more important. Further, they claimed that the time-varying weights should not introduce severe nonstationarity in the filter approximation because the variance ratios do not vary much through the time. The correlation between the filtered-out components and the theoretical ones at different leads and lags symmetrically diminishes as the leads and lags go far away, which might indicate that the degree of asymmetry was not great. Finally, a CF filter derived under the Random-Walk data generating process, the so-called Random Walk filter, gives a good approximation to the optimal filtering that explicitly used the estimated coefficients of an optimal moving average process determined empirically. Therefore, they claimed that we could use the Random Walk filter without inspecting the data generating process even if the random walk assumption was false. In the paper, we simply denote it by CF henceforth.

Details of the CF filter are given in Christiano and Fitzgerald (2003) and its properties are discussed in Iacobucci and Noullez (2005). As argued in Otsu (2015), the cyclical components extracted by CF might be distorted in magnitude and timing. The *gain* function, defined as the modulus of the frequency response function, shows large ripples over the target ranges, indicating a large distortion in estimating the cyclical components. It also shows leakage effects over higher frequencies of more than 8 periods per cycle. Further, phase shifts are indicated by values of its phase function, defined as arctangent of the ratio of the real-valued coefficient of the imaginary part of the frequency response function to the real part value.

In the paper, we first compute the components of 12 to 2-month cycles, that is, the frequency range $\left[\frac{2\pi}{12}, \frac{2\pi}{2}\right]$, and subtract them from the original series to obtain $\hat{y}_t$.

4.2 *Hamming-Windowed Filter*

Iacobucci and Noullez (2005) claimed that the Hamming-windowed filter be a good candidate for extracting frequency-defined components. The proposed filter has a flatter response over the passband than other filters in the literature, such as the HP filter (Hodrick and Prescott, 1997), the BK filter (Baxter and King, 1999), and the CF filter. This means that it has no exacerbation and eliminates high-frequency components better than the other three filters.

The Hamming-windowed filtering is implemented in the frequency domain. The procedure is described as follows. First, we subtract, if necessary, the least-square regression line to detrend the observation series to make it suitable for the Fourier transform. Second, we implement the Fourier transform of the detrended series, Third, we convolve the ideal response with a spectral window to find the windowed filter response in the frequency domain. The window is the so-called Tukey-Hamming window (Priestly, 1981, pp.433-442). In the paper, we compute the components with cycles longer than the 13-month cycle, that is, the frequency range $\left[0, \frac{2\pi}{13}\right]$, to obtain $\hat{y}_t$, the seasonally-adjusted counterparts.

4.3 *Butterworth Filters*

Pollock (2000) have proposed the tangent-based Butterworth filters in the two-sided expression, which are called rational square-wave filters. The one-sided

Butterworth filters are widely used in electrical engineering, and well documented in standard text books, such as Oppenheim and Schafer (1999) and Proakis and Manolakis (2007). The two-sided version guarantees phase neutrality or no phase shift. It has finite coefficients, and its frequency response is maximally flat over the pass band: the first $(2n-1)$ derivatives of the frequency response are zero at zero frequency for the nth-order filter. The filter could stationarize an integrated process of order up to $2n$. The order of the filter can be determined so that the edge frequencies of the pass band and/or the stop band are aligned to some designated frequencies. Further, Gomez (2001) pointed out that the two-sided Butterworth filters could be interpreted as a class of statistical models called UCARIMA (the unobserved components autoregressive-integrated moving average) in Harvey (1989, p. 74). Since the two-sided Butterworth filters are not so often used in the literature, we present relevant equations to look at them a little bit more closely.

The lowpass filter is expressed as

$$BFT_L = \frac{(1+L)^n(1+L^{-1})^n}{(1+L)^n(1+L^{-1})^n + \lambda(1-L)^n(1-L^{-1})^n} \tag{20}$$

where $L^d x_t = x_{t-d}$, and $L^{-d} x_t = x_{t+d}$. Similarly, the highpass filter is expressed as

$$BFT_H = \frac{\lambda(1-L)^n(1-L^{-1})^n}{(1+L)^n(1+L^{-1})^n + \lambda(1-L)^n(1-L^{-1})^n} \tag{21}$$

Note $BFT_L + BFT_H = 1$, which is the complementary condition discussed by Pollock (2000, p. 321). Here, λ is the so-called smoothing parameter. We observe that the Butterworth highpass filter in eq. (21) can handle nonstationary components integrated of order $2n$ or less. Let ω_c the *cutoff point* at which the gain is equal to 0.5. It is shown

$$\lambda = \{tan(\omega_c/2)\}^{-2n} \tag{22}$$

To see this, we replace the L by $e^{-i\omega}$ in eq. (20) to obtain the frequency response function in polar form as

$$\psi_L(e^{-i\omega};\lambda,n)=\frac{1}{1+\lambda(i(1-e^{-i\omega})/(1+e^{-i\omega}))^{2n}} \tag{23}$$

$$=\frac{1}{1+\lambda\{tan(\omega/2)\}^{2n}} \tag{24}$$

Here, it is easy to see that eq. (22) holds when $\psi_L(e^{-i\omega})=0.5$. We also observe in eq. (24) that the first $(2n-1)$ derivatives of $\psi_L(e^{-i\omega})$ are zero at $\omega=0$; thus, this filter is maximally flat. Note that the gain is the modulus of the frequency response function, and indicates to what degree the filter passes the amplitude of a component at each frequency. The Butterworth filters considered here are symmetric and their frequency response functions are non-negative. Therefore, the gain is equivalent to the frequency response. Then, we can use eq. (24) to specify ω_c so that the gain at the edge of the pass band is close to one and that of the stop band close to zero. Let the pass band $[0, \omega_p]$, and the stop band $[\omega_s, \pi]$, where ω_p is smaller than ω_s. As in Gomez (2001, p. 372), we consider the following conditions for some small positive values of δ_1 and δ_2,

$$1-\delta_1<|\psi_L(e^{-i\omega};\lambda,n)|\leqq 1 \quad \text{for} \quad \omega\in[0,\omega_p] \tag{25}$$

$$0\leqq|\psi_L(e^{-i\omega};\lambda,n)|<\delta_2 \quad \text{for} \quad \omega\in[\omega_s,\pi] \tag{26}$$

That is, we can control leakage and compression effects with precision specified by the values of δ_1 and δ_2. These conditions can be written as follows:

$$1+\left(\frac{tan(\omega_p/2)}{tan(\omega_c/2)}\right)^{2n}=\frac{1}{1-\delta_1} \tag{27}$$

$$1+\left(\frac{tan(\omega_s/2)}{tan(\omega_c/2)}\right)^{2n}=\frac{1}{\delta_2} \tag{28}$$

Then, we can solve for the cutoff frequency (ω_c) and the filter's order (n), given

ω_p, ω_s, δ_1 and δ_2. The closer to zeros both δ_1 and δ_2, the smaller the leakage and the compression effects. If n turns out not an integer, the nearest integer is selected.

The Butterworth filters could be based on the sine function. Instead of eq. (20) and eq. (21), the lowpass and the highpass filters can be written as follows, respectively.

$$BFS_L = \frac{1}{1+\lambda(1-L)^n(1-L^{-1})^n} \tag{29}$$

$$BFS_H = \frac{\lambda(1-L)^n(1-L^{-1})^n}{1+\lambda(1-L)^n(1-L^{-1})^n} \tag{30}$$

where

$$\lambda = \{2 sin(\omega_c/2)\}^{-2n} \tag{31}$$

These are the so-called sine-based Butterworth filters. When n is equal to two, eq. (30) is the HP cyclical filter, derived in King and Rebelo (1993, p. 224). Thus, as pointed out by Gomez (2001, p. 336), the sine-based Butterworth filter with order two $(n=2)$ can be viewed as the HP filter. As in the case of the tangent-based one, the cutoff point, ω_c, can be determined with the following conditions:

$$1+\left(\frac{sin(\omega_p/2)}{sin(\omega_c/2)}\right)^{2n} = \frac{1}{1-\delta_1} \tag{32}$$

$$1+\left(\frac{sin(\omega_s/2)}{sin(\omega_c/2)}\right)^{2n} = \frac{1}{\delta_2} \tag{33}$$

We observe that the Butterworth highpass filter in eq. (21) or eq. (30) can handle nonstationary components integrated of order $2n$ or less. Thus, the HP filter can stationarize the time series with unit root components up to the fourth order. Gomez (2001, p. 367) claimed that the BFT would give better approximations to ideal low-pass filters than the BFS. A simulation study in Otsu (2007) confirmed it.

In the following analysis, we use BFT to extract passband components $[0, \omega_p]$, setting $\omega_p = \frac{2\pi}{13}$, with the stop band $[\omega_s, \pi]$ setting $\omega_s = \frac{2\pi}{12}$. To implement the Butterworth filtering, we need specify two parameter values, n and λ, in eq. (20) or eq. (21). We obtain these values from eqs. (22), (27) and (28) for target frequency bands, that is, values of ω_p and ω_s with given values of δ_1 and δ_2. We set both δ_1 and δ_2 to 0.01. We only use BFS (2nd order) to obtain the HP-filtered passband components, setting $\omega_c = \frac{2\pi}{13}$ in eq. (31).

Turning to implementation, we can implement the Butterworth filtering either in the time domain or in the frequency domain. Following Pollock (2000), Otsu (2007) implemented it in the time domain, and found that when the cycle period is longer than seven, the matrix inversion is so inaccurate that it is impossible to control leakage and compression effects with a certain precision specified by eq. (27) and eq. (28), or eq. (32) and eq. (33). Further, the filters at the endpoints of data have no symmetry due to the finite truncation of filters. This implies that the time-domain implementation introduces phase shifts. Therefore, we do not choose the time-domain filtering.

Alternatively, we can implement the Butterworth filtering in the frequency domain. In the frequency-domain filtering, cyclical components are computed via the inverse discrete Fourier transform, using the Fourier-transformed series with the frequency response function as their weights. In contrast to the time-domain filtering, the frequency-domain filtering does not introduce any phase shifts, as the theoretical background of the symmetrical filters dictates. For the frequency-domain procedures to work well, it is required that a linear trend be removed and circularity be preserved in the time series, which we discuss next.

4.4 *Detrending Method*

To obtain better estimates of cyclical components, it is desirable to remove a linear trend in the raw data. The linear regression line, recommended by Iacobucci and Noullez (2005), is often used for trend removal. As shown by Chan, Hayya, and Ord (1977) and Nelson and Kang (1981), however, this method can produce spurious periodicity when the true trend is stochastic. Another widely-used detrending method is the first differencing, which reweighs toward the higher frequencies and can distort the original periodicity, as pointed out by Baxter and King (1999), Chan, Hayya, and Ord (1977), and Pedersen (2001).

Otsu (2011a) found that the drift-adjusting method employed by Christiano and Fitzgerald (2003, p. 439) could preserve the shapes of autocorrelation functions and spectra of the original data better than the linear-regression-based detrending. Therefore, this detrending method would create less distortion. Let the raw series z_t, $t=1, \cdots, T$. Then, we compute the drift-adjusted series, x_t, as follows:

$$x_t = z_t - (t+s)\hat{\mu} \tag{34}$$

where s is any integer and

$$\hat{\mu} = \frac{z_T - z_1}{T-1} \tag{35}$$

Note that the first and the last points are the same values:

$$x_1 = x_T = \frac{Tz_1 - z_T + s(z_1 - z_T)}{T-1} \tag{36}$$

In Christiano and Fitzgerald (2003, p. 439), s is set to -1. Although Otsu (2011a) suggested some elaboration on the choice of s, it does not affect the results of our subsequent analyses in the paper. Thus, we also set s to -1.

It should be noted that the drift-adjusting procedure in eq. (34) would make the data suitable for filtering in the frequency domain. Since the discrete Fourier transform assumes circularity of data, the discrepancy in values at both ends of the time series could seriously distort the frequency-domain filtering. The eq. (36) implies that this adjustment procedure avoids such a distortionary effect.

A final remark here is that the BB procedure is implemented with trend-included series. In the business cycle literature, it is important to distinguish a classical cycle and a growth one, as pointed out by Pagan (1997). The classical cycle consists of peaks and troughs in the *levels* of aggregate economic activities, often represented by the gross national product (GDP). The classical cycle is studied by Burns and Mitchell (1946), one of the influential seminal works, which found that business cycles range from 18 months (1.5 years) to 96 months (8 years) for the United States.

On the other hand, the growth cycle exists in the *detrended* series, on which the real business cycle literature focuses. The two types of the cycles show different dates of the peaks and the troughs. When a series has a cyclical component around a deterministic upward trend, typical as in economic data, detrending would make the peaks earlier, while delaying the troughs (see Bry and Boschan, 1971, p. 11). For this reason, the dating based on the growth cycle generically tends to deviate from that on the classical cycle. Then, Canova (1994, 1999) judged that the estimated dates matched the official dates as long as deviations were within two or three quarters. The results in Otsu (2013) also show that the estimated dates of peaks based on the detrended series tend to mark earlier and those of troughs later than the official dates. Since we only suppress the cyclical components shorter than the seasonal cycle in the paper, we do not have such a deviation due to detrending.

4.5 Boundary Treatment

In addition to the detrending method, we make use of another device to reduce variations of the estimates at ends of the series: extension with a boundary treatment. As argued by Percival and Walden (2000, p. 140), it might be possible to reduce the estimates' variations at endpoints if we make use of the so-called *reflection boundary treatment* to extend the series to be filtered. We modify the *reflection boundary treatment* so that the series is extended antisymmetrically instead of symmetrically as in the conventional reflecting rule. Let the extended series f_j,

$$f_j = \begin{cases} x_j & \text{if } 1 \leqq j \leqq T \\ 2x_1 - x_{2-j} & \text{if } -T+3 \leqq j \leqq 0 \end{cases} \tag{37}$$

That is, the $T-2$ values, folded antisymmetrically about the initial data point, are appended to the beginning of the series. We call this extension rule the *antisymmetric reflection*, distinguished from the conventional reflection.

It is possible to append them to the end of the series. The reason to append the extension at the initial point is that most filters give accurate and stable estimates over the middle range of the series. When we put the initial point in the middle part of the extended series, the starting parts of the original series would have estimates more robust to data revisions or updates than the ending parts. Since the initial data point indicates the farthest past in the time series, it does not make sense that the estimate of the initial point is subject to a large revision when additional observations are obtained in the future. Otsu (2010) observed that it moderately reduced compression effects of the Butterworth and the Hamming-windowed filters. We note that this boundary treatment makes the estimates at endpoints identically zero when a symmetric filter is applied. We filter the extended series, f_j, and extract the last T values to obtain the targeted components, that is, seasonal adjustment factors that are subtracted from the original series to obtain the seasonally-adjusted series.

5 Empirical Analysis

5.1 Reference Dates and Data

The reference dates of business cycles in Japan are determined by Economic and Social Research Institute (ESRI), affiliated with the Cabinet Office, Government of Japan. ESRI organizes the Investigation Committee for Business Cycle Indicators to inspect historical diffusion indexes calculated from selected series of coincident indexes and other relevant information. To make a historical diffusion index, the peaks and troughs of each individual time series are dated by the Bry-Boschan method. Thus, the reference dates correspond to those of peaks and troughs of the classical cycles, that is, the Burns-and-Mitchell-type cycle based on the level of aggregate economic activity. Typically, the final determination of the dates is made about two to three years later.

Table 3 shows the reference dates of peaks and troughs identified by ESRI. It also contains periods of expansion, contraction, and duration of a complete cycle (trough to trough). There are 15 peak-to-trough phases identified after World War II. The average period is about 36 months for expansion, 16 for contraction, and 52 for the complete cycle. We compare the reference dates with those of the growth cycles obtained by filtering methods.

ESRI routinely examines and revises composition of the indicators. Although the latest revision is made in February 2017, our data are based on the 9th revision in November 2004, adopted until September 2011, that selected 11 economic series for the coincident indicators. We use 11 composite coincident indicators of Japan in monthly basis, retrieved from Nikkei NEEDS CD-ROM (2008). Series names, as well as mnemonics, are listed in Table 2. The sample period ranges from January 1980 to January 2008, 337 observations for each series. We choose this data set for two reasons. First, it gives a fairly long time series in consistent composition of the indicators.

Secondly, it is revealed that officials at Ministry of Health, Labor and Welfare have incorrectly conducted fundamental statistical survey on labor-related conditions since 2004. Then, one of the 11 series, 'Index of Non-Scheduled Worked Hours,' may need correction. This issue is being under investigation as of 27th January, 2019. Our data may include possibly incorrect data for four years after 2004. It is desirable for the following analyses not only to include business cycles as many as possible but to avoid contaminated data as much as possible. This consideration leads us to focus on the sample up to January 2008 based on the 9th-revision composition.

We note that among the series, 'Operating Profits' is available only in quarterly base (end of periods) with seasonal adjustment (X12-ARIMA). We linearly extrapolate the quarterly data points to make monthly series. All the index-type data have the base year in 2000.

5.2 Aggregation of Coincident Indicators

To examine to what extent the composite coincident index (CCI) deviates from the reference cycle, we compare the official reference dates with the dates of peaks and troughs implied by the CCI. We use the Bry-Boschan algorithm procedure (see section 3) to identify dates of peaks and troughs of the CCI, because ESRI uses it to calculate the diffusion index that gives fundamental information to determine the reference dates. Here, we use the coincident indicators seasonally adjusted by the official agents, so that we can exclude influence of seasonal adjustment on dating results.

In the first ('Official Ref. Dates') and the second ('Official CCI') columns of Table 4, we find that dates of peaks differ between the official reference cycle and the official CCI, except May 1997. The official CCI identifies November 1981 as a peak, while the official peak date indicates February 1980. Since the composition of coincident indicators is routinely revised, the set of indicators used in 1980 is different from that of the paper. This would be one reason for the discrepancy. Yet, there are other reasons as well.

As already mentioned, ESRI uses the historical diffusion indices (coincident indicators) to determine the reference cycle. However, only publicly available are the materials used at the committee after 2002 onward. Thus, we alternatively use the current diffusion index (Nikkei NEEDS CD-ROM, 2008) to examine the deviation between the reference cycle and the composite indices. We find that the index took 92 on average during February 1979 to February 1981. This might give rise to the official peak date, February 1980. During May 1980 to May 1981, the current diffusion index took less than 50 points. It reached 54.5 in June 1981, marked 100 in August, then down to 54.5 in December and to less than 50 afterwards. The official CCI seems pick up these small bumps. There would be two reason for this discrepancy. First, the Bry-Boshcan algorithm uses the moving averages and the Spencer smoothing: the former become asymmetric at endpoints and the latter uses averages of the initial or the last four points as observations at endpoints. Thus, it may introduce distortion in dating computation. Secondly, it eliminates peaks within 6 months at endpoints in Step IV (see Table 1). Therefore, it never identifies February 1980 as a peak since our data set begins from January 1980. Then, we do not pay much attention to the deviation from the reference dates in early 1980s in the following analysis.

To check our aggregation program, we attempt to replicate the official CCI by aggregating the coincident indicators (seasonally adjusted series) published by the official agents, according to the procedure described in section 2. Note that the first quartile in eq. (5), $Q1_i^j$, is set to the 84th value of 336 rates of changes, $r_i^j(t)$ in eq. (1), in ascending order, and the third quartile, $Q3_i^j$, to the 253d value. The results are shown in Table 4 and Table 5. The dates in the third column ('Aggr. Indicators') in Table 4 match well with the dates given by the official CCI. Only difference is observed in 1981. In the fourth column ('Aggr. Ind. (No Trim)'), we do not implement the outlier removal (the threshold value: 2.02) to see its effect. Then, the peak in 1997 becomes two months earlier, March instead of May. Although we find similar effects later in the paper, it is fair to say that the outlier removal has only a limited role in dating the reference cycle.

As for the troughs, the results are shown in Table 5. the official CCI deviates from the reference dates in 1993 by two months and in 1999 by one month. The aggregation of the coincident indicators gives dating results similar to the official CCI with or without outlier removal. The deviation from the reference dates is 6 months for the peaks from 1985 to 2000 and 3 months for the troughs from 1983 to 2002. A large deviation in the troughs is observed for the aggregated index, but this is mainly due to the deviation in the early 1980s: February 1983 versus October 1982. If we exclude it, the deviation reduces to 3 months. Therefore, it can be said that the computed index yields the dating results equivalent to those that the official CCI does.

We now examine whether the dates of the turning points depend on the location and the scaling measures in aggregation of coincident indicators. ESRI uses the 5-year averages defined in eq. (3) and the interquartile ranges to standardize the rate of change of each indicators in eq. (5). These quantities seem preferred because they are supposed to be insensitive to outliers. The third columns in Table 4 and in Table 5 give the results of the corresponding case. When we use the sample mean instead of the 5-year trend, we have the results in the second ('Interquartile' of 'Sample Mean') and the third ('Standard Deviation' of 'Sample Mean') columns in Table 6 and Table 7. These results are very similar to those the official CCI gives in Table 4 and Table 5. It is interesting to see that use of the sample mean makes the dating results closer to those of the official CCI, and that whether we use the interquartile ranges or the standard deviation does not matter. In addition, the fourth column ('Standard Deviation' of 'CM: 5-year Trend') in Table 7 indicates that use of the 5-year trend introduces the deviation from the date of February 1983. Here, we have no evidence to encourage the use of the interquartile ranges and the 5-year trend.

5.3 Smoothing Methods and Bry-Boshcan Procedure

ESRI uses seasonally adjusted series. The conventional seasonal adjustment attempts to remove seasonal frequencies only, leaving all the higher frequencies in the series. In terms of economic analysis, there is no sound reason that economic data should include the frequencies higher than seasonal ones. If an economic theory neither presumes seasonality or difference of seasonality among economic variables nor designates specific forms of econometric models with seasonal effects, it is most likely not to intend to explain the fluctuation shorter than seasonality. Further, if the main concern is about the business cycle, we may remove such a shorter cyclical movement in data.

In this section, we use filtering methods discussed in section 4, instead of the X12-ARIMA method, to remove all the frequencies higher than seasonality. Each of the 11 coincident indicators is filtered and aggregated to make a composite coincident index. Otsu (2011b) examined the performance of the filtering methods used in the paper to extract the seasonal components, and found that they are very useful and the corresponding 'seasonally-adjusted' series are smoother than the series with the X12-ARIMA seasonal adjustment.

In Table 8 and Table 9, we use the tangent-based Butterworth filter. The results of 'Case 1' are obtained with the Bry-Boschan (BB) procedure, skipping the steps of I, III and IV in Table 1. We also note that these results are exactly same as those of the full BB procedure. This implies that the outlier removal and the Spencer smoothing have no effect on the results. The columns of 'Case 2' show the results when we further remove Step V.1-V.2 in the BB procedure. Thus, the data processing in the BB procedure is limited to the asymmetric 12-month moving average. As for the peaks, the dates fit better with the official reference dates, whether we use the sample mean or the 5-year trend, while we observe not much improvement for dating the troughs. The deviation is 3 months for the peaks, and 7 months for the troughs. This contrasts with the results of the official seasonally-adjusted data in Table 4 and Table 5, in which the dates of the troughs show smaller deviations than those of the peaks.

In experiments, we confirm that the choice of the scaling measures, the interquartile ranges or the standard deviation, has no influence on the results. Again, neither the central measures nor the scaling measures does not play an important role. In sum, the simple mean and the standard deviation is good enough, and the 12-month moving average with the simplified BB procedure gives dating results that are close enough to the official reference dates, when we use the Butterworth filter for the seasonal adjustment.

The Hamming-windowed filter produces similar results as the Butterworth filter. The second column of 'Case 1' in Table 10 only shows two-month deviation in 1991. Interestingly, when we use the interquartile range as a scaling measure, the result gets closer to those of the second column in Table 8. This implies that the Butterworth filter works as the interquartile range does. Although the equivalent result of 'Case 1' is also obtained with the full BB procedure, skipping Step V.1 and V.2 in Table 1 gives a different result to the third column of 'Case 2', but it is quite similar to the result in the corresponding column in Table 8. Only difference between the third columns in both tables is in that choice of the scaling measures has some effects on the dating result in case of the Hamming-windowed filter. In the fourth and the fifth column, we find that use of the 5-year trend does not show much improvement in dating. The results of the troughs in Table 11 are also equivalent to those in Table 9, specifically in the third column, the ‘Case 2’ of the sample mean. Again, we cannot find clear evidence for the 5-year trend, the interquartile ranges, and the Spencer smoothing.

Turning to the Christiano-Fitzgerald filter, we find in the columns of ‘Case 1’ in Table 12 and Table 13 that the dating results depend on the choice of the scaling measures. Using the interquartile ranges makes the dating results closer to those of the Butterworth filter in Table 8 and Table 9. In contrast, the results of ‘Case 2’ are less likely to be susceptible to the scaling choice, and comparable to those of the Butterworth filter. The deviation tends to be slightly larger, but it is fair to say that it is same in magnitude. Finally, the Hodrick-Prescott filter gives rise to the results that depend on the choice of the scaling measures and that get closer to those of the Butterworth when the series are standardized with the interquartile ranges. The deviation tends to be larger compared with the results of other filtering methods.

To investigate the effects of the outlier removal with a threshold value of 2. 02, we have repeated the same calculation of Table 8 through Table 15 with the composite indices computed with the outlier removing process. The main difference is found in the latter half of 1990s. When we use the sample mean and the standard deviation as the location and the scaling measures, respectively, we obtain a peak in May 1997 with all the filtering methods but the HP filter, and a trough in February 1999 with all the filters. The former is exactly same as that of the reference cycle, and the latter is different from the reference date by only one month. Thus, the outlier removal plays a certain role in the late 1990s.

In our final experiment, we use the composite coincident indices complied with the filtered indicators, instead of the 12-month moving average (MA12), in Step II of Table 1, so that the BB procedure becomes free from phase shifting.

Table 16 and Table 17 show the results. First, we find new dates of peaks and troughs in 1995. The peaks are dated in January with the Butterworth, the Hamming-windowed, and the CF filters, and in March with the HP filter. Since we had the Great *Hanshin-Awaji* Earthquake in 17th Janurary, 1995, it is reasonable to identify January 1995 as a peak. The troughs are identified in July with the Butterworth and the Hamming-windowed filters, June with the CF filter, and August with the HP filter. It is certainly arguable that these dates should be suppressed. Secondly, we find that these results are identical to those of ‘Case 1’ in Table 8 through Table 15, except 1995. Thus, it can be interpreted that phase-shift effects by the MA12 are offset by the Step V. 1 and V. 2 in Table 1. Third, implementing the outlier removal with the threshold value of 2.02 makes a slight change in the results in 1986 and 1999. It is observed that the dates in these years get closer to the reference dates.

In short, when we use either the Butterworth or the Hamming-windowed filters, the dating results do not depend on the choice of the central and the scaling measures. On the other hand, the Christiano-Fitzgerald filter and the Hodrick-Prescott filter produce the dating results that are susceptible to the choice, in particular, of the scaling measures. Use of the interquartile ranges makes the dating results closer to those of the Butterworth filter. In addition, the Spencer smoothing and the 12-month moving average in the BB procedure does not play much role when the filtering methods are used. Further, it does no harm to skip the final steps in the BB procedure to ensure peaks and troughs within the short-term periods (Step V.1 and V.2 in Table 1), specifically with the Butterworth filter. Finally, we note that we have equivalent results over the different filtering methods when we use the sample mean as a central measure and skip Step I, III through V.2.

6 Discussion

This paper attempts to find an alternative method to make the coincident composite index that is simpler than the existing method and is consistent with the official reference cycle as much as possible. We examine how accurately the coincident composite index trace the reference cycle, using the business cycle data of Japan. Further, we investigate aggregation methods to construct the composite index from the individual indicators in terms of normalization of each indicator, smoothing methods, and dating algorithm.

The main findings are as follows. First, although ESRI uses an outlier trimming procedure and interquartile ranges for a scaling measure, these play only a minor role in dating peaks and troughs, possibly, in the late 1990s. Secondly, when filtering methods is used to seasonally adjust series, the Bry-Boschan algorithm can be simplified to a great extent and does not need the Spencer smoothing, the 12-month moving averages, and a trimming procedure. Thirdly, the Butterworth filter has the same effects on dating results as the interquartile ranges. Therefore, it reduces variation of the series good enough.

These findings suggest that it is possible to simplify the compilation process of the composite indices. Since the composite indices await various kinds of economic analyses for vairous purpose, simplicity and clarity in the compilation is desirable. Although ESRI uses X12-ARIMA for seasonal adjustment, it involves arbitrariness in selecting ARIMA models, setting parameters, judgements on statistical significance of estimates. Moreover, different economic variables require different X12-ARIMA models. Then, X12-ARIMA could distort the relation among economic variables, as pointed out by Sims (1974) and Wallis (1974). In contrast, the filtering methods only require frequencies or periods per cycle to be preserved or suppressed, which might be given by economic analyses, and make it possible to filter any variable with exactly same parameter values.

When we use the Butterworth or the Hamming-windowed filter to seasonally adjust series, we do not have to use the 5-year trend as a location measure to standardizing the economic indicators. It is not clear why we should use the 5-year trend. The judgement is rather subjective. Further, the filtering also reduces the necessity of the interquartile ranges as a scaling measure, indicating that the standard deviation works well. The threshold value is set as arbitrarily as the conventional significance levels in statistics, such as 10%, 5%, and 1%. We also find that the outlier removal seems not play an important role. There is no firm grounds that we should remove 5% at edges of the standardized series. and that In many cases, the so-called outliers in economic variables give some clues to understand important economic phenomena or effects of exogenous variables. Then, careless outlier removal would be harmful. These findings indicate that we could use a simple frequency-domain filtering and a conventional normalization to construct a composite coincident index.

Concerning the Bry-Boschan algorithm, we can get rid of the Spencer and the moving-average smoothing, so that we may avoid arbitrariness accompanied by the former in determination of polynomial orders and phases shifts introduced by the latter. All what we need is to determine the minimum duration of phases and cycles and the enforcement rules of alternation of peaks and troughs. We do not need such repeated processes as in the original BB procedure to determine the dates of peaks and troughs.

In the paper, we have found and discussed a possibility of simplifying the aggregation and the dating algorithm to identify peaks and troughs of the business cycle. However, we need to investigate samples in other periods and other countries to come to a final conclusion. Further, we may need to investigate other dating rules suggested in the literature (see Webb, 1991; Harding and Pagan, 2016).

These are left for the future research.

Acknowledgements **A part of this work is financially supported by** *Tokubetsu Kenkyu Josei* **(Seijo University) under the title of**「景気動向指数の作成法と頑健手法に関する研究」.

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Table 1 Summary of Bry-Boschan Procedure

Step	Procedure
I	Outlier-removed series (XO): The data point of the original series (X) is replaced by that of the Spencer-filtered series (XSP) if its normalized difference in absolute value is larger than or equal to 3.5.
II	Dating with 12-month moving average: 1. Moving average: Compute 12-month moving average with 6 lags and 5 leads (X12), using XO. 2. Identification of peaks and troughs: Find the maximum (peak) or the minimum (trough) of X12 values within 6-month leads and lags. 3. Enforcement of alternation: Ensure the peaks and the troughs are alternate. If not, choose a peak with a greater value and a trough with a smaller value. If the values are same, choose an earlier peak and a later trough.
III	Dating with Spencer filtering: 1. Spencer filtering: Filtering XO with the Spencer filter to obtain a series named XOSP. 2. Identification of peaks and troughs: Ensure the peaks and the troughs as in Step II within 6 months, using XOSP. Modify if necessary. 3. Enforcement of alternation: Ensure alternation as in Step II. 4. Enforcement of minimum cycle duration: Check if the duration of a peak-to-peak or trough-to-trough takes at least 15-month period. If not, choose higher peaks and lower troughs, or if equal, an earlier date for a peak and a later one for a trough.
IV	Dating with short-term moving average: 1. Spencer filtering: Use the Spencer curve of the original series (X) as the trend-cycle component, and compute the irregular component by the difference between X and the Spencer curve. Find the minimum number of months (MCD, Months of Cyclical Dominance) over which the average rate of change in the trend-cycle component exceeds the average change in the irregular component. If it is less than 3 months, the MCD is set to 3, while set to 6 if more than 6 months. 2. Short-term moving average: Compute the short-term moving average (MCDX) of the original series (X) with the span of MCD obtained above. The values at the first and the last dates with missing values in leads and lags, are to set to the same values as those at the nearest dates. 3. Identification of peaks and troughs: Ensure the peaks and the troughs as in Step III within 6 months, using MCDX series. Modify if necessary. 4. Enforcement of alternation: Check alternation as in Step II.
V	Dating with the original series: 1. Identification of peaks and troughs: Ensure the peaks and the troughs as in Step IV within 4 months or MCD, whichever longer, using the original series (X). Modify if necessary. 2. Enforcement of alternation: Ensure alternation as in Step II. 3. Elimination of turns within 6 months at endpoints: Eliminate peaks and troughs within 6 months of beginning and end of series. 4. Enforcement of the first and last peak (or trough) to be extrema: Eliminate peaks (or troughs) at both ends of series which are lower (or higher) than values closer to end. 5. Enforcement of the minimum cycle duration: Check if the peak-to-peak and the trough-to-trough cycles are less than 15 months. If not, eliminate lower peaks (or higher troughs), or if equal, a later peak and an earlier trough. 6. Enforcement of the minimum phase duration: Eliminate phases (peak to trough or trough to peak) whose duration is less than 5 months.

Table 2 Coincident Indicators: Japan (9th Revision: Nov. 2004 - Sept. 2011)

Series Name	Mnemonic (NEEDS)*
1. Index of Industrial Production (Mining and Manufacturing)	IIP00P001(@)
2. Index of Producer's Shipments (Producer Goods for Mining and Manufacturing)	IIP00S255(@)
3. Large Industrial Power Consumption, mil. kwh.	CELL9(@)
4. Index of Capacity Utilization Ratio (Manufacturing)	IIP00O01(@)
5. Index of Non-Scheduled Worked Hours (Manufacturing)	HWINMF00 (HWINMF05@)
6. Index of Producer's Shipment (Investment Goods Excluding Transport Equipment)	IIP00S204 (IIP00SINV@)
7. Retail Sales Value (Change From Previous Year, %)	ZCSHVB20 (ZCSHVB20V)
8. Wholesale Sales Value (Change From Previous Year, %)	ZCSHVB00 (ZCSHVB00V)
9. Operating Profits, thou. mil. yen (All Industries)	ZBOAS@**
10. Index of Sales in Small and Medium Sized Enterprises (Manufacturing)	SMSALE@
11. Effective Job Offer Rate (Excluding New School Graduates)	ESRAO(@)

* "@" indicates seasonally-adjusted series.

** Only quarterly series are available. A linear-interpolation is used to obtain monthly series.

Table 3 Reference Dates of Business Cycles in Japan

Dates (month, year)		Number of Periods (in months)		
Peak	Trough	Expansion	Contraction	Duration
June, 1951	October, 1951	—	4	—
January, 1954	November, 1954	27	10	37
June, 1957	June, 1958	31	12	43
December, 1961	October, 1962	42	10	52
October, 1964	October, 1965	24	12	36
July, 1970	December, 1971	57	17	74
November, 1973	March, 1975	23	16	39
January, 1977	October, 1977	22	9	31
February, 1980	February, 1983	28	36	64
June, 1985	November, 1986	28	17	45
February, 1991	October, 1993	51	32	83
May, 1997	January, 1999	43	20	63
November, 2000	January, 2002	22	14	36
February, 2008	March, 2009	73	13	86
March, 2012	November, 2012	36	8	44

Source: *Indexes of Business Conditions*, Economic and Social Research Institute, Cabinet Office, Government of Japan, July 24, 2015.

Table 4 Comparison with Reference Dates: Peaks, Official S.A. Series

Official Ref. Dates		Official CCI		Aggr. Indicators*		Aggr. Ind. (No Trim)**	
Year	Month	Year	Month	Year	Month	Year	Month
1980	2						
		1981	11	1981	12	1981	12
1985	6	1985	5	1985	5	1985	5
		1990	10	1990	10	1990	10
1991	2						
1997	5	1997	5	1997	5	1997	3
2000	11	2000	12	2000	12	2000	12
2008	2						
Deviation***		6 months		6 months		8 months	

Note: * Compiled from coincident indicators (S.A. series).
** Outlier removal (threshold value: 2.02) is not implemented.
*** Deviation from the reference dates, sum of absolute values from 1985 to 2000.

Table 5 Comparison with Reference Dates: Troughs, Official S.A. Series

Official Ref. Dates		Official CCI		Aggr. Indicators*		Aggr. Ind. (No Trim)**	
Year	Month	Year	Month	Year	Month	Year	Month
1977	10						
		1981	5	1981	5	1981	5
				1982	10	1982	10
1983	2	1983	2				
1986	11	1986	11	1986	11	1986	11
1993	10	1993	12	1993	12	1993	12
		1998	12	1998	12	1998	12
1999	1						
2002	1	2002	1	2002	1	2002	1
2009	3						
Deviation***		3 months		7 months		7 months	

Note: * Compiled from coincident indicators (S.A. series).
** Outlier removal (threshold value: 2.02) is not implemented.
*** Deviation from the reference dates, sum of absolute values from 1983 to 2002.

Table 6　Comparison with Reference Dates: Peaks, Alternative Measures in eq.(5)

Official Ref. Dates		Central Measure(CM): Sample Mean				CM : 5-year Trend, eq.(3)	
		Interquartile		Standard Deviation		Standard Deviation	
Year	Month	Year	Month	Year	Month	Year	Month
1980	2						
		1981	11	1981	11	1981	11
1985	6	1985	5	1985	5	1985	5
		1990	10	1990	10	1990	10
1991	2						
1997	5	1997	5	1997	5	1997	5
2000	11	2000	12	2000	12	2000	12
2008	2						
Deviation*		6 months		6 months		6 months	

Note: 1. May (1997) is altered to March with no outlier removal (threshold value: 2.02).
2. Same results without Step I, III & IV and Spencer smoothing in BB proc. (Table 1)
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

Table 7　Comparison with Reference Dates: Troughs, Alternative Measures in eq.(5)

Official Ref. Dates		Central Measure(CM): Sample Mean				CM: 5-year Trend, eq.(3)	
		Interquartile		Standard Deviation		Standard Deviation	
Year	Month	Year	Month	Year	Month	Year	Month
1977	10						
		1981	5	1981	5	1981	5
						1982	10
1983	2	1983	2	1983	2		
1986	11	1986	11	1986	11	1986	11
1993	10	1993	12	1993	12	1993	12
		1998	12	1998	12	1998	12
1999	1						
2002	1	2002	1	2002	1	2002	1
2009	3						
Deviation*		3 months		3 months		7 months	

Note: 1. Same results are obtained with or without outlier removal (threshold value: 2.02).
2. Same results without Step I, III & IV and Spencer smoothing in BB proc.(Table 1)
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.

Table 8 Variants of BB Procedure: Peaks, Butterworth (tangent based) Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1980	2								
		1981	9	1981	11	1981	9	1981	12
1985	6	1985	7	1985	6	1985	7	1985	6
		1990	11			1990	11	1990	12
1991	2			1991	1				
1997	5	1997	4	1997	4	1997	4	1997	5
2000	11	2000	11	2000	10	2000	11	2000	10
2008	2								
Deviation*		5 months		3 months		5 months		3 months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1985 to 2000.

Table 9 Variants of BB Procedure: Troughs, Butterworth (tangent based) Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1977	10								
		1981	2	1981	4	1981	1		
				1982	12			1982	12
1983	2	1983	1			1983	1		
1986	11	1986	11	1986	10	1986	11	1986	10
1993	10	1993	12	1993	12	1993	12	1993	12
1999	1	1999	4	1999	3	1999	4	1999	2
		2001	12			2001	12		
2002	1			2002	1			2002	1
2009	3								
Deviation*		7 months		7 months		7 months		6 months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* Deviation from the reference dates, sum of absolute values from 1983 to 2002.

Table 10　Variants of BB Procedure: Peaks, Hamming-Windowed Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1980	2								
		1981	10	1981	11	1981	10	1981	12
1985	6	1985	6(7)*	1985	6	1985	7	1985	6
		1990	12(11)*			1990	11	1990	12
1991	2			1991	1				
1997	5	1997	5	1997	4	1997	5	1997	5
2000	11	2000	11	2000	10	2000	11	2000	10
2008	2			(2007	6)*			(2007	6)*
Deviation**		2(4) months		3 months		4 months		3 months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when scaling measure is the interquartile range.
** Deviation from the reference dates, sum of absolute values from 1985 to 2000.

Table 11　Variants of BB Procedure: Troughs, Hamming-Windowed Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1977	10								
		1981	2	1981	4	1981	2	(1981	3)*
				1982	12			1982	12
1983	2	1983	2			1983	2		
1986	11	1986	10	1986	10	1986	10	1986	10
1993	10	1993	12	1993	12	1993	12	1993	12
1999	1	1999	4	1999	3	1999	4	1999	2
		2001	12			2001	12		
2002	1			2002	1			2002	1
2009	3								
Deviation**		7 months		7 months		7 months		6 months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when scaling measure is the standard deviation.
** Deviation from the reference dates, sum of absolute values from 1983 to 2002.

Table 12 Variants of BB Procedure: Peaks, Christiano-Fitzgerald Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1980	2								
		1981	8(9)*	1981	11	1981	9	1981	12
1985	6	1985	6(7)*	1985	6	1985	6(7)*	1985	6
		1990	10	1990	12	1990	10	1990	12
1991	2								
1997	5	1997	5(4)*	1997	4	1997	5	1997	5
2000	11	2000	10(11)*	2000	10	2000	10(11)*	2000	10
2008	2								
Deviation**		5(6) months		4 months		5(5) months		3 months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when scaling measure is the interquartile range.
** Deviation from the reference dates, sum of absolute values from 1985 to 2000.

Table 13 Variants of BB Procedure: Troughs, Christiano-Fitzgerald Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1977	10								
		1981	1	1981	4	1981	1		
		1982	12	1982	12	1982	12	1982	12
1983	2								
1986	11	1986	11	1986	10	1986	11	1986	10
1993	10	1993	12	1993	12			1993	12
						1994	1		
1999	1	1999	3(4)*	1999	3	1999	3(4)*	1999	2(3)*
		2001	12			2001	12		
2002	1			2002	1			2002	1
2009	3								
Deviation**		7(8) months		7 months		8(9) months		6(7) months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when scaling measure is the interquartile range.
** Deviation from the reference dates, sum of absolute values from 1983 to 2002.

Table 14　Variants of BB Procedure: Peaks, Hodrick-Prescott Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1980	2								
		1981	10	1981	12	1981	11		
								1982	1
1985	6	1985	6(7)*	1985	6	1985	7	1985	6
		1990	10			1990	10	1990	12
1991	2			1991	1**				
1997	5	1997	4(3)*	1997	4	1997	4(3)*	1997	5
2000	11	2000	10	2000	9(10)*	2000	10	2000	10
2008	2			2007	6			2007	6
Deviation***		6(8) months		4(4) months		7(8) months		3 months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when scaling measure is the interquartile range.
** The date of December 1990 is obtained instead of January 1991, when scaling measure is the interquartile range.
*** Deviation from the reference dates, sum of absolute values from 1985 to 2000.

Table 15　Variants of BB Procedure: Troughs, Hodrick-Prescott Filter

Official Ref. Dates		Central Meas.: Sample Mean				CM: 5-year Trend, eq.(3)			
		Case 1		Case 2		Case 1		Case 2	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1977	10								
		1981	2	1981	4				
				1982	12			1982	11(12)*
1983	2	1983	1			1983	1		
1986	11	1986	11	1986	10	1986	11	1986	10
1993	10	1993	12	1993	12	1993	12	1993	12
1999	1	1999	5	1999	2(3)*	1999	1(5)*	1999	2(3)*
		2001	12			2001	12		
2002	1			2002	1			2002	1
2009	3								
Deviation**		8 months		6(7) months		4(8) months		7(9) months	

Note: Case 1 Skip I, III and IV in BB procedure (Table 1).
Case 2 Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when scaling measure is the interquartile range.
** Deviation from the reference dates, sum of absolute values from 1983 to 2002.

Table 16 Bry-Boschan Procedure without Internal Smoothing: Peaks

Official Ref. Dates		Butterworth		Hamming		CF Filter		HP Filter	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1980	2								
		1981	9	1981	10	1981	8	1981	10
1985	6	1985	7	1985	6	1985	6	1985	7(6)*
		1990	11	1990	12	1990	10	1990	10
1991	2								
		1995	1	1995	1	1995	1	1995	3
1997	5	1997	4	1997	5	1997	5	1997	4
2000	11	2000	11	2000	11	2000	10	2000	10

Note: 1. Sample mean for central measure, standard deviation for scaling measure.
2. Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when outliers are removed (threshold of 2.02).

Table 17 Bry-Boschan Procedure without Internal Smoothing: Troughs

Official Ref. Dates		Butterworth		Hamming		CF Filter		HP Filter	
Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.	Year	Mon.
1977	10								
		1981	2	1981	2	1981	1	1981	2
						1982	12		
1983	2	1983	1	1983	2			1983	1
1986	11	1986	11	1986	10	1986	11	1986	11
1993	10	1993	12	1993	12	1993	12	1993	12
		1995	7	1995	7	1995	6	1995	8
1999	1	1999	4(3)*	1999	4(3)*	1999	3	1999	5(1)*
		2001	12	2001	12	2001	12	2001	12
2002	1								

Note: 1. Sample mean for central measure, standard deviation for scaling measure.
2. Skip I and III through V.2 in BB procedure (Table 1).
* The number in ( ) shows a month when outliers are removed (threshold of 2.02).